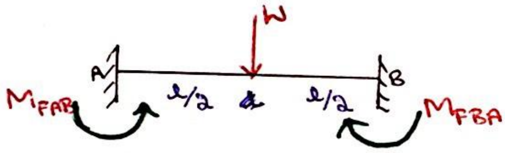
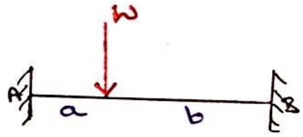
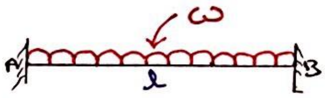
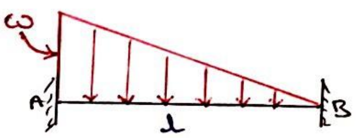
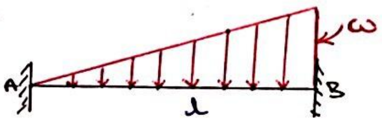
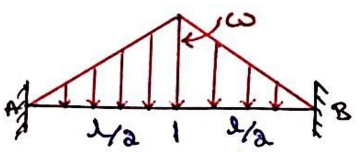
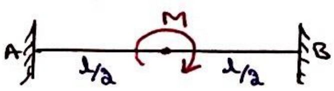
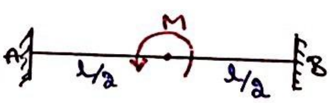
Sunil.H.G

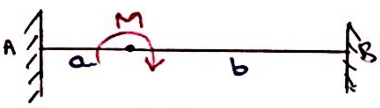
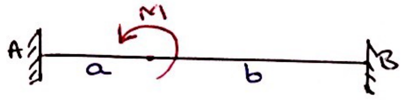
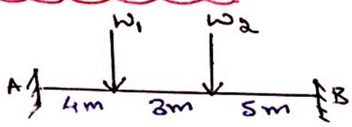
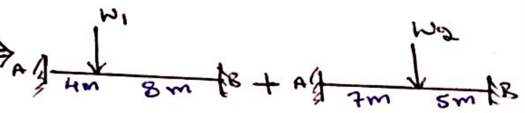
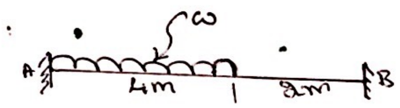
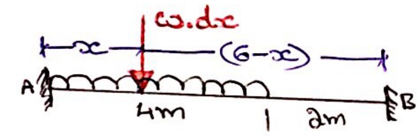
MODULE-1

SLOPE - DEFLECTION METHOD

(*) Procedure:
1) FEM
2) S.D. equations
3) Equilibrium Conditions
4) Final Moments
5) Diagrams (SFD, BMD & EC)

1) FEM (Fixed End Moments)

SL No.	Load pattern	M_{FAB}	M_{FBA}
1.		$-\frac{Wl}{8}$	$+\frac{Wl}{8}$
2.		$-\frac{Wab^2}{l^2}$	$+\frac{Wa^2b}{l^2}$
3.		$-\frac{\omega l^2}{12}$	$+\frac{\omega l^2}{12}$
4.		$-\frac{\omega l^2}{20}$	$+\frac{\omega l^2}{30}$
5.		$-\frac{\omega l^2}{30}$	$+\frac{\omega l^2}{20}$
6.		$-\frac{5\omega l^2}{96}$	$+\frac{5\omega l^2}{96}$
7.		$+\frac{M}{4}$	$+\frac{M}{4}$
8.		$-\frac{M}{4}$	$-\frac{M}{4}$

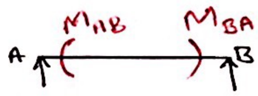
Sl. No.	Load pattern	M_{FAB}	M_{FBA}
9.		$+\frac{Mb}{l^2}(2a-b)$	$+\frac{Ma}{l^2}(2b-a)$
10.		$-\frac{Mb}{l^2}(2a-b)$	$-\frac{Ma}{l^2}(2b-a)$
	Other Cases:		
11.			
	⇒ 	$-\left[\frac{W_1ab^2}{l^2}+\frac{W_2ab^2}{l^2}\right]$	$+\left[\frac{W_1a^2b}{l^2}+\frac{W_2a^2b}{l^2}\right]$
12.			
	⇒ 	$-\frac{wab^2}{l^2}$ $=-\int_0^4\frac{(\omega.dx)(x)(6-x)^2}{l^2}$	$+\frac{wa^2b}{l^2}$ $=+\int_0^4\frac{(\omega.dx)(x)^2(6-x)}{l^2}$

(*) Overhang Portion ⟶ No FEM

2) Slope-Deflection equation

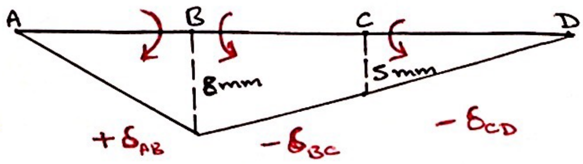
$$M_{AB}=\frac{2EI}{l}\left[2\theta_A+\theta_B-\frac{3\delta}{l}\right]+M_{FAB}$$

$$M_{BA}=\frac{2EI}{l}\left[2\theta_B+\theta_A-\frac{3\delta}{l}\right]+M_{FBA}$$



(*) @ Fixed Support ⟶ $\theta=0$
No Sinking (or) Non-sway ⟶ $\delta=0$

(*) write rotation Symbol (↻) @ the max. deflection point. Always the Arrow head should be below the line.



Sign: (↻ +) (↺ −)

δ-Value: Difference b/w Deflection @ the ends.

(*) Overhang portion ⟶ No S-D equation.

3> Equilibrium Conditions:

→ @ intermediate Support joint : $\Sigma M = 0$
→ @ Last joint (Simple, Roller (or) Hinged Support) : $M = 0$

4> Final Moments :

→ Substitute 'θ' values in S-D equation and get the Final Moments.

(*) Overhang portion ⟶ Calculate Final Moment directly.

5> Diagrams (SFD, BMD & EC) :

(a) SFD:

→ Draw FBD –

- Write given beam line, points & distances.
- Write given loads
- Put vertical reaction at all the supports.
- Write the final moments.

→ Calculate Support Reactions –

$\Sigma H = 0$
$\Sigma V = 0$
& $\Sigma M = 0$

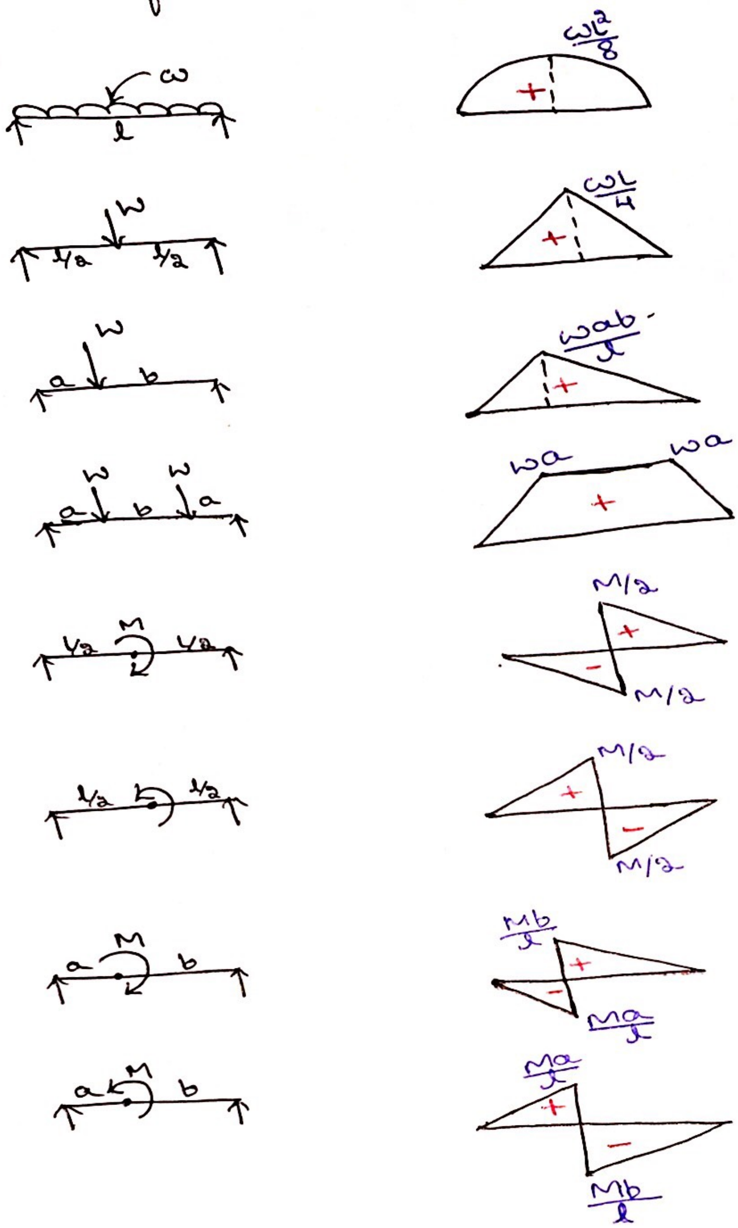
→ Write SFD.

(b) BMD :

BMD ⟶ (Free BMD + Final BMD)

NOTE:
(*) Simply Supported beams – @ ends : $M = 0$
(*) For Special load pattern, Consider the Beam portion Seperately & Calculate moments @ intermediate points & draw BMD.
(*) Final BMD – Draw a line at the tail side of the rotation arrow & get Final BMD.

(*) BMD for standard load cases.

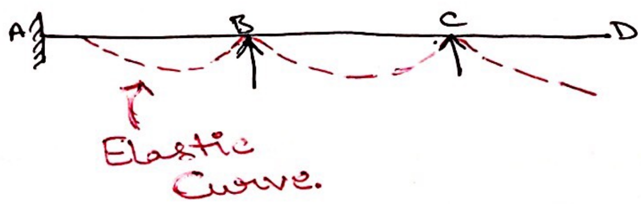


(c) Elastic Curve:

(*) Fixed end → Dont join the line directly

other Supports → join the line directly.

Overhang → Dont close the Curve.



Date
24/08/08

Structural Analysis - II

(a) Sign Convention

(1) Reaction

$\Sigma V = 0$, ↑ +ve ↓ −ve

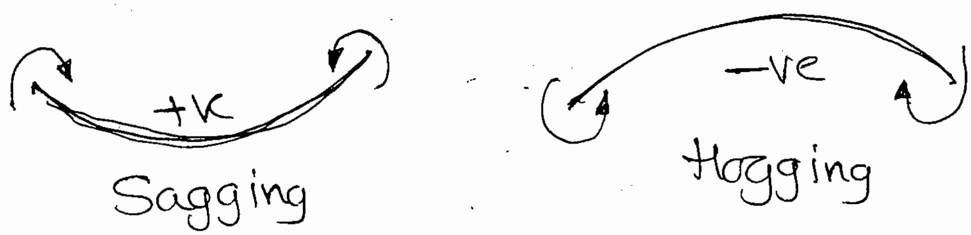
$\Sigma H = 0$, → +ve ← −ve

$\Sigma M = 0$, +ve ↻ ↺ −ve

(2) Shear Force

From "Left" to "Right" ↑ +ve ↓ −ve

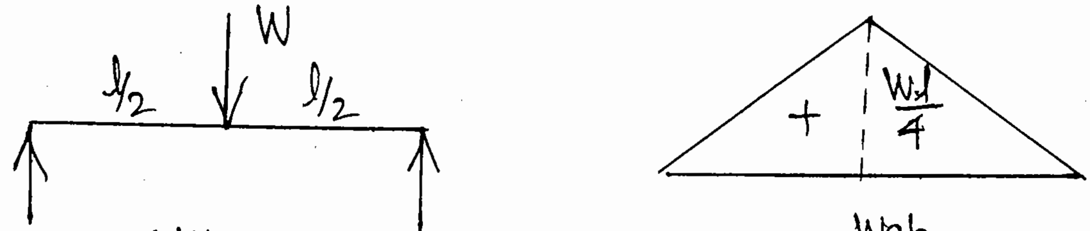
(3) Bending Moment

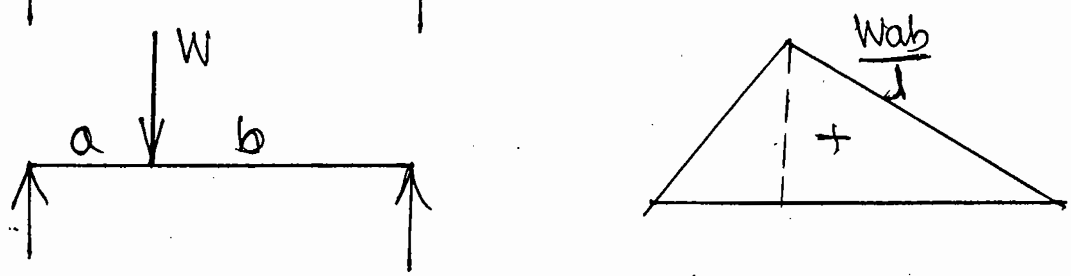


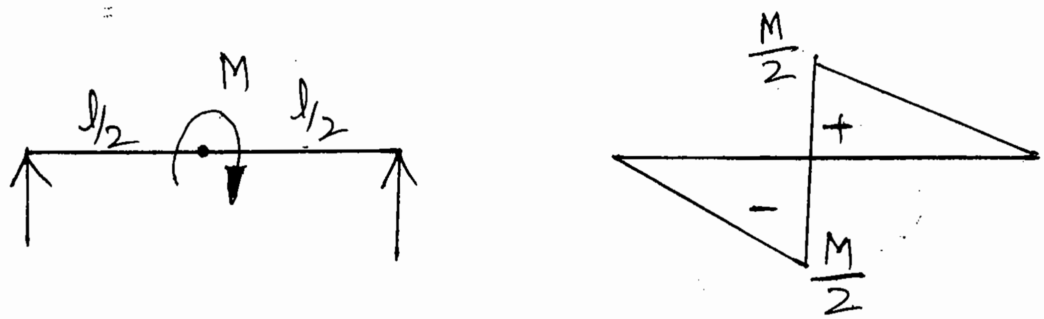
Or Clockwise Moment → +ve

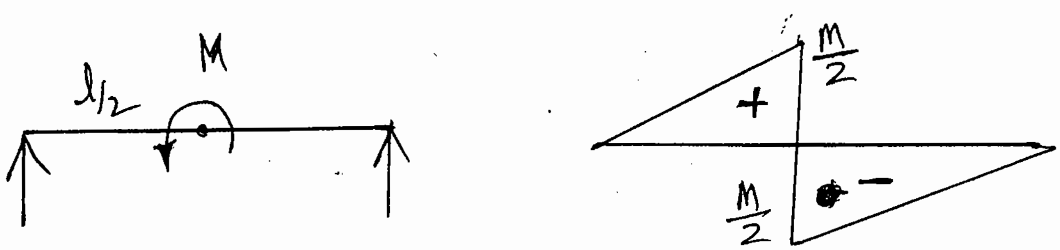
Anti-clockwise " → −ve

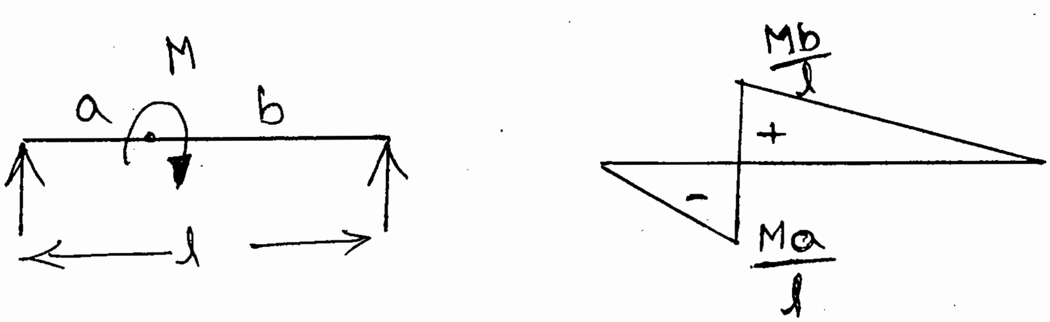
(b) BMD: (Free BMD)



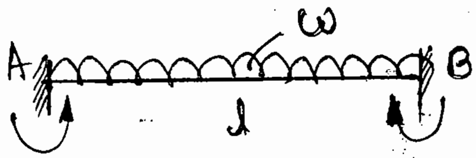




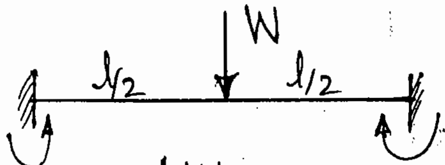




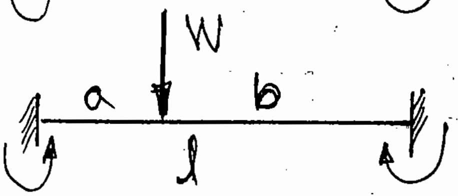
(c) Fixed End Moments :-



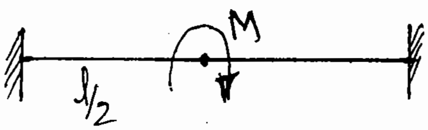
$$M_{FAB} = -\frac{\omega l^2}{12}, \quad M_{FBA} = +\frac{\omega l^2}{12}$$



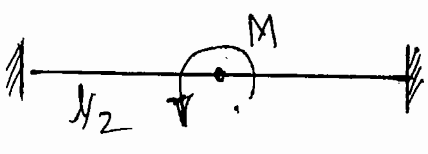
$$M_{FAB} = -\frac{Wl}{8}, \quad M_{FBA} = +\frac{Wl}{8}$$



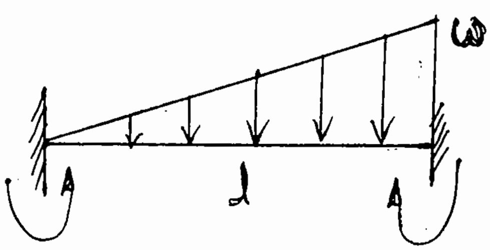
$$M_{FAB} = -\frac{Wab^2}{l^2}, \quad M_{FBA} = +\frac{Wa^2b}{l^2}$$



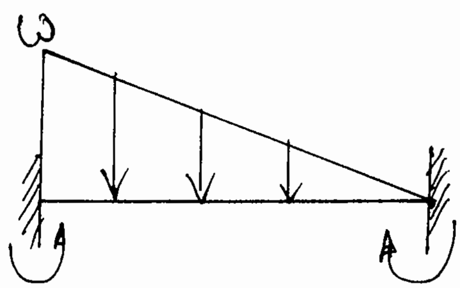
$$M_{FAB} = M_{FBA} = +\frac{M}{4}$$



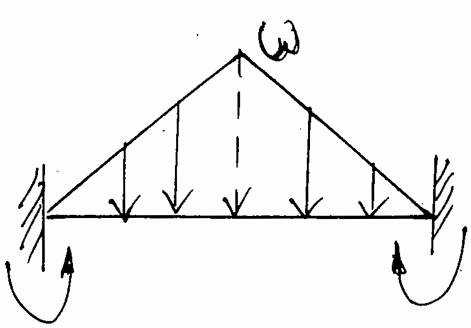
$$M_{FAB} = M_{FBA} = -\frac{M}{4}.$$



$$M_{FAB} = -\frac{\omega l^2}{30}, \quad M_{FBA} = +\frac{\omega l^2}{20}$$



$$M_{FAB} = -\frac{\omega l^2}{20}, \quad M_{FBA} = +\frac{\omega l^2}{30}$$



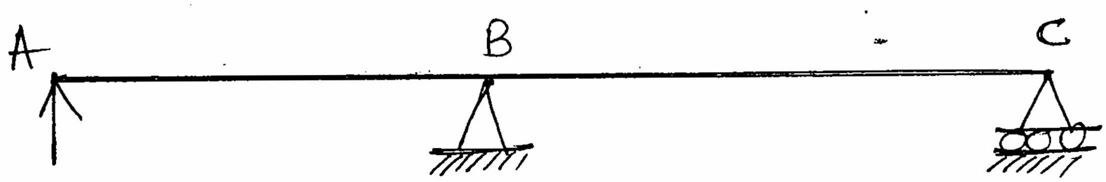
$$M_{FAB} = -\frac{5\omega l^2}{96} \quad M_{FBA} = +\frac{5\omega l^2}{96}$$

(d) (1) Intermediate Support :

If there is a span on both sides of supports then it is called "intermediate" support

∴ "B" & "C" → Intermediate

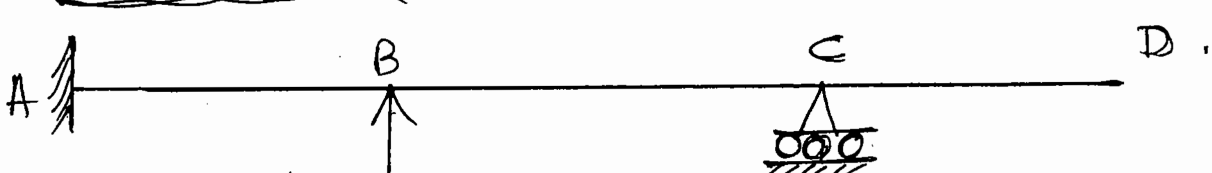
(2) Last Support :



If span is on one side only, then it is called last simple or hinge or Roller support.

∴ At last support $\boxed{\text{Moment} = 0}$

(3) Equillibrium Condition :-

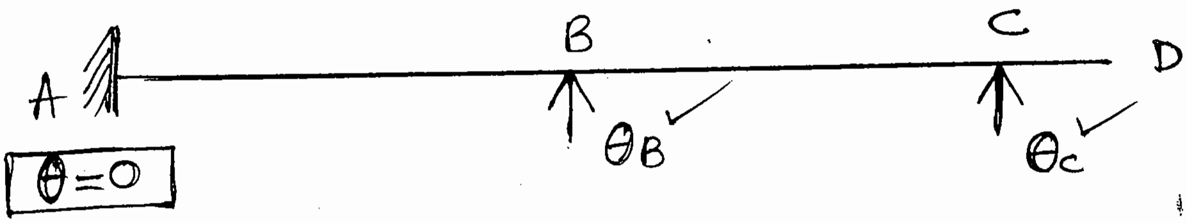


At intermediate support, $\boxed{\text{Sum of Moment} = 0}$

At "B" $\boxed{M_{BA} + M_{BC} = 0}$

At "C" $\boxed{M_{CB} + M_{CD} = 0}$

(I) Slope Deflection Method :



Basic Equation

$$M_{AB} = \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right] + M_{FAB}$$

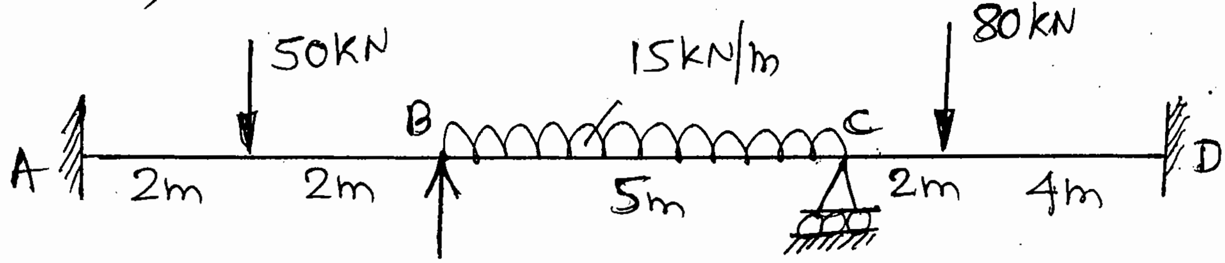
$$M_{BA} = \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right] + M_{FBA}$$

$\theta \rightarrow$ Slope or Rotation

$\delta \rightarrow$ Sinking or Settlement

$EI \rightarrow$ Flexural Rigidity.

Eg:- 1] Analyse the continuous beam shown by S.D. method and draw BMD; SFD and EC.



EI → Constant

Sol^n

(a) FEM

$$M_{FAB} = -\frac{Wl}{8} = \frac{-50\times4}{8} = -25\,kN\text{-}m$$

$$M_{FBA} = +\frac{Wl}{8} = +25\,kN\text{-}m$$

$$M_{FBC} = -\frac{\omega l^2}{12} = -\frac{15\times5^2}{12} = -31.25$$

$$M_{FCB} = +\frac{\omega l^2}{12} = +31.25$$

$$M_{FCD} = -\frac{Wab^2}{l^2} = -\frac{80\times2\times4^2}{6^2} = -71.11\,kN\text{-}m$$

$$M_{FDC} = +\frac{Wa^2b}{l^2} = \frac{80\times2^2\times4}{6^2} = +35.56\,kN\text{-}m$$

(b) S.D. Equation:

$\theta_A = \theta_D = 0$ ($\because$ Fixed Support)

$\delta = 0$ ($\because$ No sinking)

$$M_{AB} = \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right] + M_{FAB}$$

$$M_{AB} = \frac{2EI}{4}\left[0 + \theta_B - 0\right] - 25 = 0.5EI\theta_B - 25 \quad \text{—(i)}$$

$$M_{BA} = \frac{2EI}{4}\left[2\theta_B + 0 - 0\right] + 25 = EI\theta_B + 25 \quad \text{—(ii)}$$

$$M_{BC} = \frac{2EI}{5}\left[2\theta_B + \theta_C - 0\right] - 31.25$$

$$= 0.8EI\theta_B + 0.4EI\theta_C - 31.25 \quad \text{—(iii)}$$

$$M_{CB} = \frac{2EI}{5}\left[2\theta_C + \theta_B - 0\right] + 31.25$$

$$= 0.8EI\theta_C + 0.4EI\theta_B + 31.25 \quad \text{—(iv)}$$

$$M_{CD} = \frac{2EI}{6}\left[2\theta_C + 0 - 0\right] - 71.11 = 0.667EI\theta_C - 71.11 \quad \text{—(v)}$$

$$M_{DC} = \frac{2EI}{6}\left[0 + \theta_C - 0\right] + 35.56 = 0.333EI\theta_C + 35.56 \quad \text{—(vi)}$$

(c) Equillibrium Condition :-

(1) At "B" $\boxed{M_{BA} + M_{BC} = 0}$ → Intermediate support

$$\left[EI\theta_B + 25\right] + \left[0.8EI\theta_B + 0.4EI\theta_C - 31.25\right] = 0$$

$$1.8EI\theta_B + 0.4EI\theta_C = 6.25 \rightarrow \text{(I)}$$

(2) At "c" $\boxed{M_{CB} + M_{CD} = 0}$

$$\left[0.8EI\theta_C + 0.4EI\theta_B + 31.25\right] + \left[0.667EI\theta_C - 71.11\right] = 0$$

$$0.4EI\theta_B + 1.467EI\theta_C = 39.86 \rightarrow \text{(II)}$$

Solving

$$\theta_B = -2.73/EI$$

$$\theta_C = +27.91/EI$$

(d) <u>Final Moments</u> [Substitute 'θ' values in eq^n (i) to (vi)]

$M_{AB} = -26.36$ kN-m

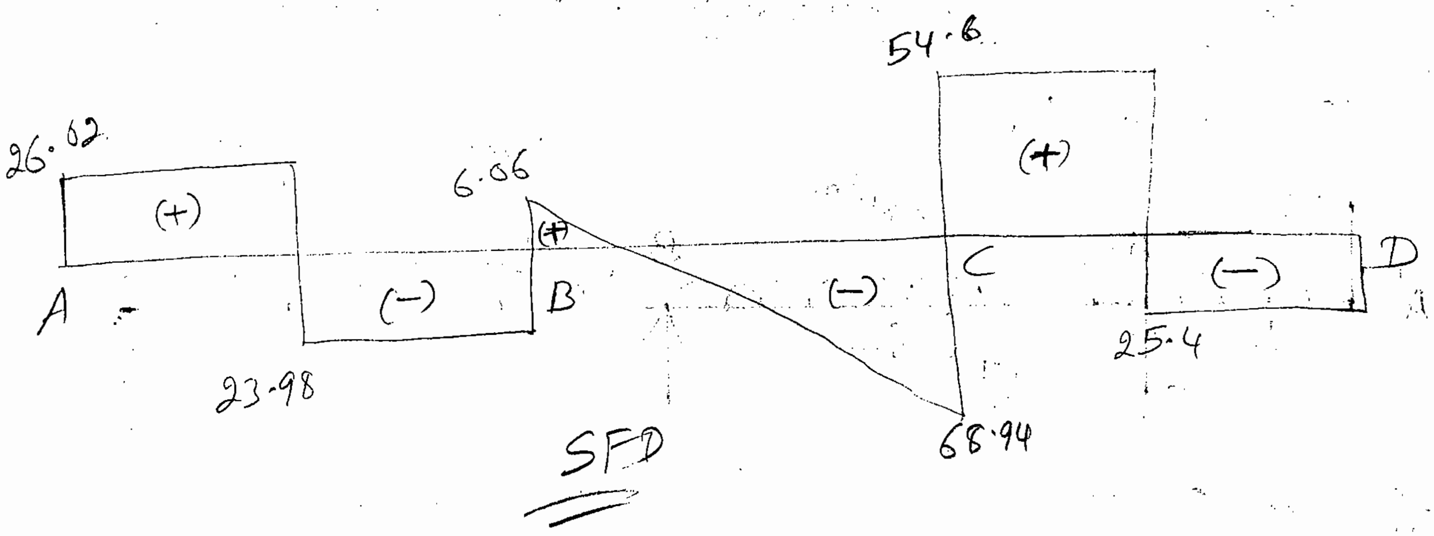
$M_{CB} = 52.48$ kN-m

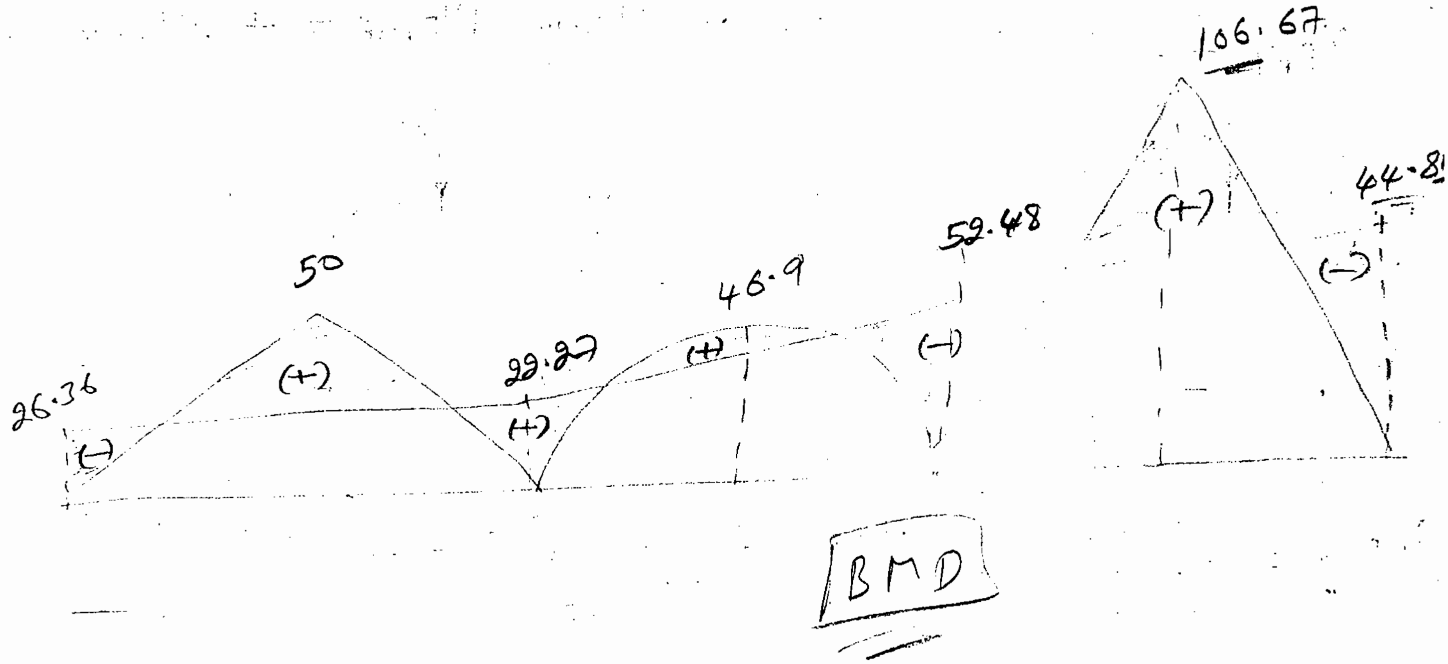
$M_{BA} = 22.27$ kN-m

$M_{CD} = -52.49$ kN-m

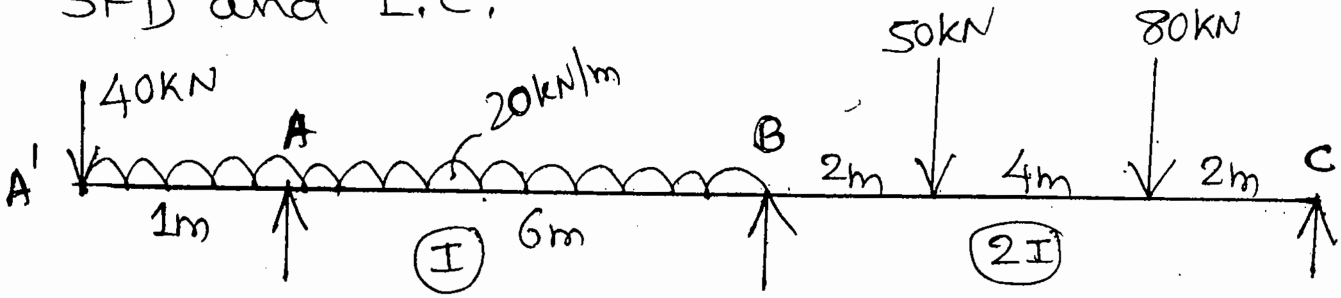
$M_{BC} = -22.27$ kN-m

$M_{DC} = 44.85$ kN-m


54.6
26.62
6.06
(+)
(+)
(+)
A
(−)
B
(−)
C
(−)
D
25.4
23.98
68.94
SFD

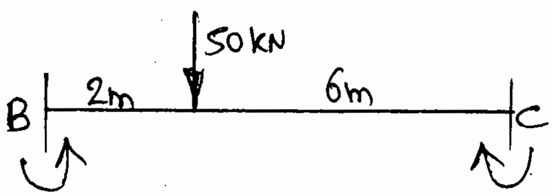

106.67
44.81
52.48
50
46.9
(+)
(−)
22.27
26.36
(+)
(+)
(−)
(−)
(+)
BMD

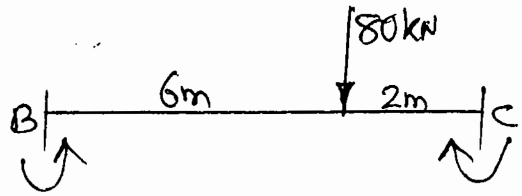
Eg:-2) Analyse the continuous beam shown by S.D. method. And draw BMD, SFD and E.C.



Sol^n (a) FEM

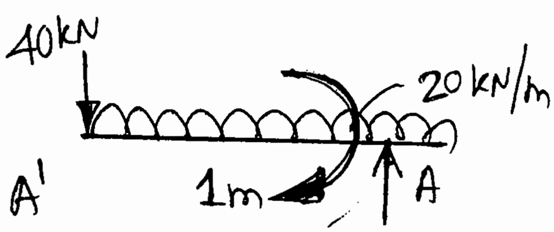
$$M_{FAB} = -\frac{wl^2}{12} = -\frac{20\times 6^2}{12} = -60\,kN\text{-}m, \quad M_{FBA} = +60\,kN\text{-}m$$





$$M_{FBC} = -\frac{Wab^2}{l^2} = -\left[\frac{50\times 2\times 6^2}{8^2} + \frac{80\times 6\times 2^2}{8^2}\right] = -86.25\,kN\text{-}m$$

$$M_{FCB} = +\frac{Wa^2b}{l^2} = +\left[\frac{50\times 2^2\times 6}{8^2} + \frac{80\times 6^2\times 2}{8^2}\right] = +108.75\,kN\text{-}m$$



$$M_{AA'} = +\left[40\times 1 + 20\times 1\times \tfrac{1}{2}\right]$$

$$= +50\,kN\text{-}m$$

+ve sign for clockwise resisting moment.

(b) S.D. Equation

$\delta = 0$ (No sinking)

$$M_{AB} = \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right] + M_{FAB}$$

$$M_{AB} = \frac{2(1 \times EI)}{6}\left[2\theta_A + \theta_B - 0\right] - 60$$

$$= (0.667EI)\theta_A + (0.333EI)\theta_B - 60 \quad \text{— (i)}$$

$$M_{BA} = \frac{2(1 \times EI)}{6}\left[2\theta_B + \theta_A - 0\right] + 60$$

$$= (0.667EI)\theta_B + (0.333EI)\theta_A + 60 \quad \text{— (ii)}$$

$$M_{BC} = \frac{2(2EI)}{8}\left[2\theta_B + \theta_C - 0\right] - 86.25$$

$$= EI\theta_B + 0.5EI\theta_C - 86.25 \quad \text{— (iii)}$$

$$M_{CB} = \frac{2(2EI)}{8}\left[2\theta_C + \theta_B - 0\right] + 108.75$$

$$= EI\theta_C + 0.5EI\theta_B + 108.75 \quad \text{— (iv)}$$

★ **Note:-** There is **NO** S.D. Equation for "overhang"

(c) Equillibrium condition :-

(i) At "A" $\boxed{M_{AA'} + M_{AB} = 0}$

$$[50] + [0.667EI\theta_A + 0.333EI\theta_B - 60] = 0$$

$$(0.667EI)\theta_A + (0.333EI)\theta_B = 10 \rightarrow \text{(I)}$$

(ii) At "B" $\boxed{M_{BA} + M_{BC} = 0}$

$$(0.333\,EI)\theta_A + (1.667\,EI)\theta_B + (0.5\,EI)\theta_C = 26.25 \rightarrow \text{(II)}$$

(iii) At "C" $\boxed{M_{CB} = 0}$ ($\because$ last Simple or Hinge or Roller support)

$$(0.5\,EI)\theta_B + (1\,EI)\theta_C = -108.75 \longrightarrow \text{(III)}$$

Solving

$$\theta_A = \frac{-15.197}{EI} \qquad \theta_C = \frac{-138.98}{EI}$$

$$\theta_B = \frac{60.47}{EI}$$

(d) Final Moment:

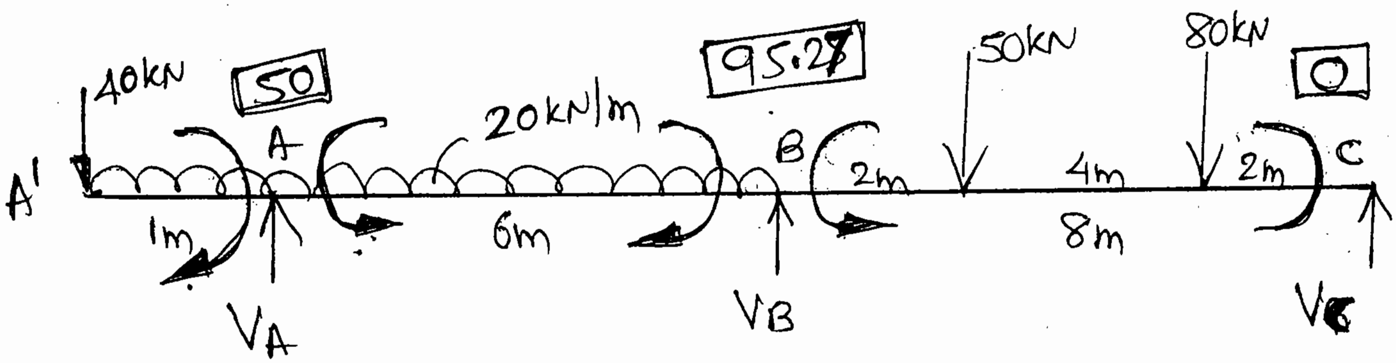
$M_{AB} = -50$ kN-m

$M_{BA} = 95.27$ kN-m

$M_{BC} = -95.27$ "

$M_{CB} = 0$

$M_{AA'} = +50$ kN-m



$$\Sigma V = 0, \quad V_A + V_B + V_C = 40 + 50 + 80 + 20 \times 7 = 310 \quad \text{--(i)}$$

$$\Sigma M_B = 0 \text{ (RHS)} - V_C \times 8 + 50 \times 2 + 80 \times 6 - 95.27 = 0$$

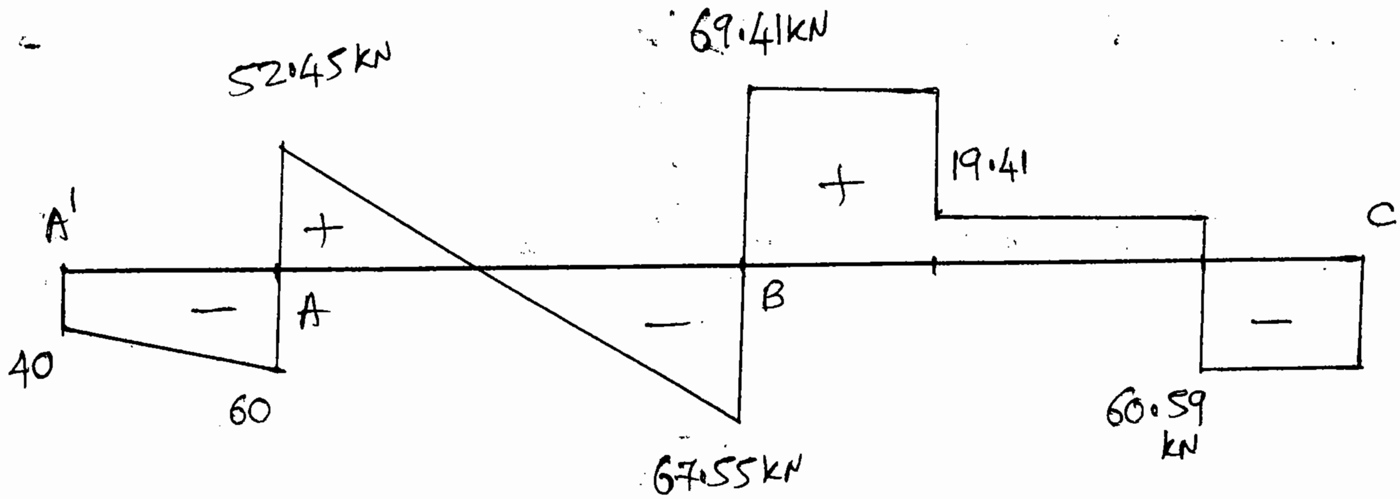
$$V_C = 60.59 \text{ kN}$$

$\Sigma M_B = 0$ (LHS)

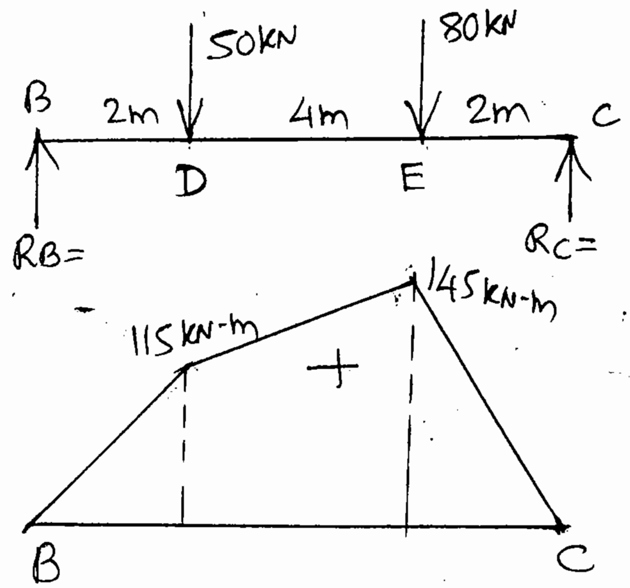
$$V_A \times 6 - 40 \times 7 - 20 \times 7 \times 7/2 + 50 - 50 + 95.27 = 0$$

$$V_A = 112.45 \text{ kN}$$

From(i) $112.45 + V_B + 60.59 = 310$ $\therefore V_B = 136.96$



Free BMD for "BC"



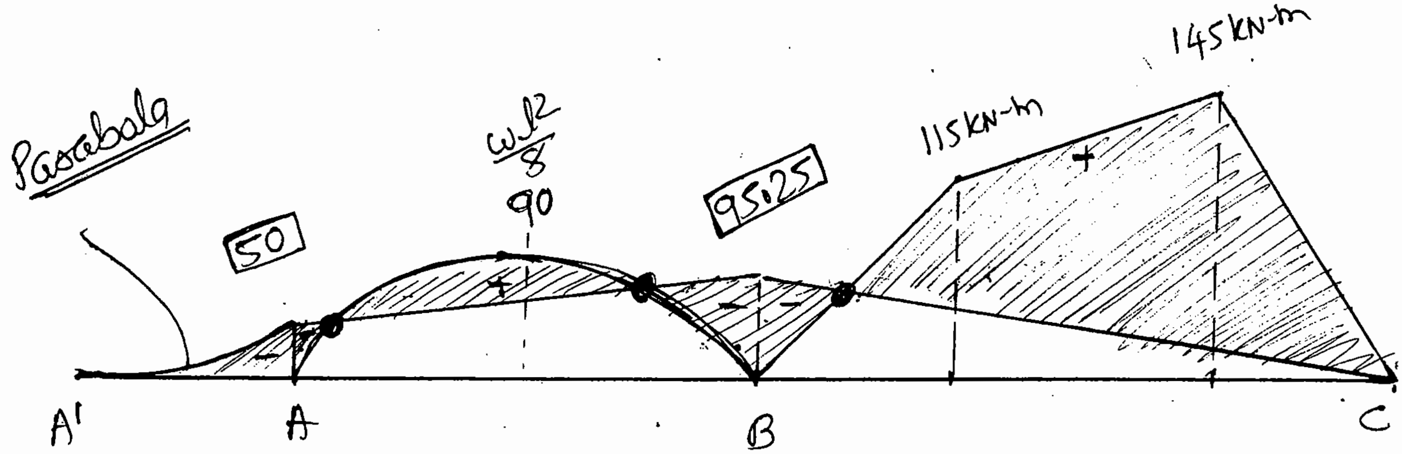
Reaction

$R_C = 72.5$ kN

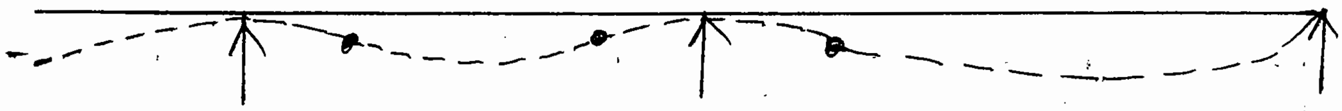
$R_B = 57.5$ kN

$\therefore M_D = R_B \times 2 = 115$ kN-m

$M_E = R_C \times 2 = 145$ kN-m



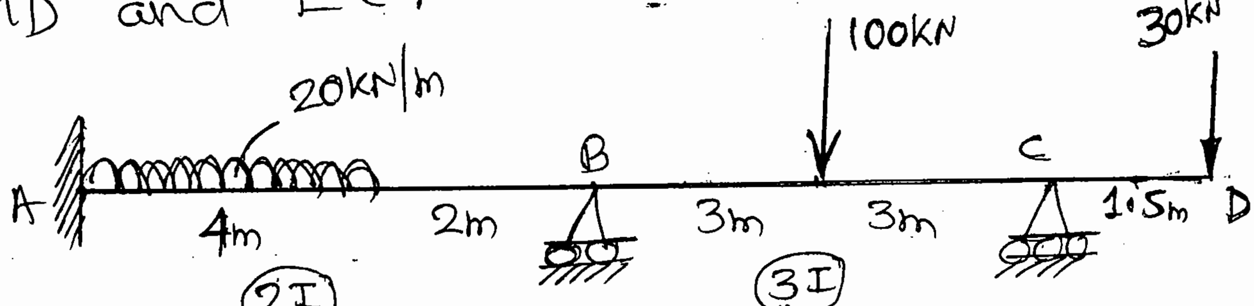
EC



EC

$=\!=\!= \times =\!=\!=$

Eg:- 3 Analyse the continuous beam shown by S.D. method Draw SFD, BMD and EC.

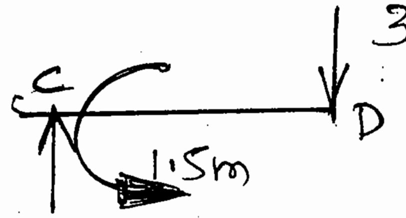


Sol²

(a) FEM

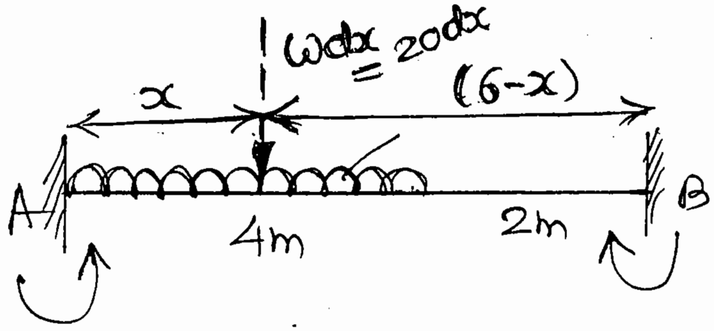
$$M_{FBC} = -\frac{Wl}{8} = -\frac{100 \times 6}{8} = -75 \text{ kN-m}$$

$$M_{FCB} = +\frac{Wl}{8} = +75 \text{ kN-m}$$



$M_{CD} = -30 \times 1.5 = \underline{\underline{-45 \text{ kN-m}}}$

–ve sign for "Anticlockwise" Resisting Moment



$W = w \cdot dx = 20dx$

$a = x$

$b = (6-x)$

$l = 6\text{m}$

$$M_{FAB} = -\frac{Wab^2}{l^2} = -\int_0^4 \frac{(20dx)(x)(6-x)^2}{6^2} = \underline{\underline{-53.33 \text{ kN-m}}}$$

$$M_{FBA} = +\frac{Wa^2b}{l^2} = +\int_0^4 \frac{(20dx)(x)^2(6-x)}{6^2} = \underline{\underline{+35.56 \text{ kN-m}}}$$

(b) S.D. Equation :

$\theta_A = 0$ ($\because$ fixed)

$\delta = 0$ ($\because$ No sinking)

For overhang there is NO SD equation

$$M_{AB} = \frac{2(2EI)}{6}\left[0 + \theta_B - 0\right] - 53.33$$

$$= 0.667 EI\theta_B - 53.33 \quad \text{— (i)}$$

$$M_{BA} = \frac{2(2EI)}{6}\left[2\theta_B + 0 - 0\right] + 35.56$$

$$= 1.333 EI\theta_B + 35.56 \quad \text{— (ii)}$$

$$M_{BC} = \frac{2(3EI)}{6}\left[2\theta_B + \theta_C - 0\right] - 75$$

$$= 2EI\theta_B + EI\theta_C - 75 \quad \text{— (III)}$$

$$M_{CB} = \frac{2(3EI)}{6}\left[2\theta_C + \theta_B - 0\right] + 75$$

$$= 2EI\theta_C + EI\theta_B + 75 \longrightarrow \text{(IV)}$$

(c) <u>Equillibrium Condition</u>

(i) At "B" $\boxed{M_{BA} + M_{BC} = 0}$

$$3.333EI\theta_B + EI\theta_C = 39.44 \longrightarrow \text{(I)}$$

(ii) At 'C' $\boxed{M_{CB} + M_{CD} = 0}$

$$EI\theta_B + 2EI\theta_C = -30 \longrightarrow \text{(II)}$$

Solving $\theta_B = 19.21/EI$

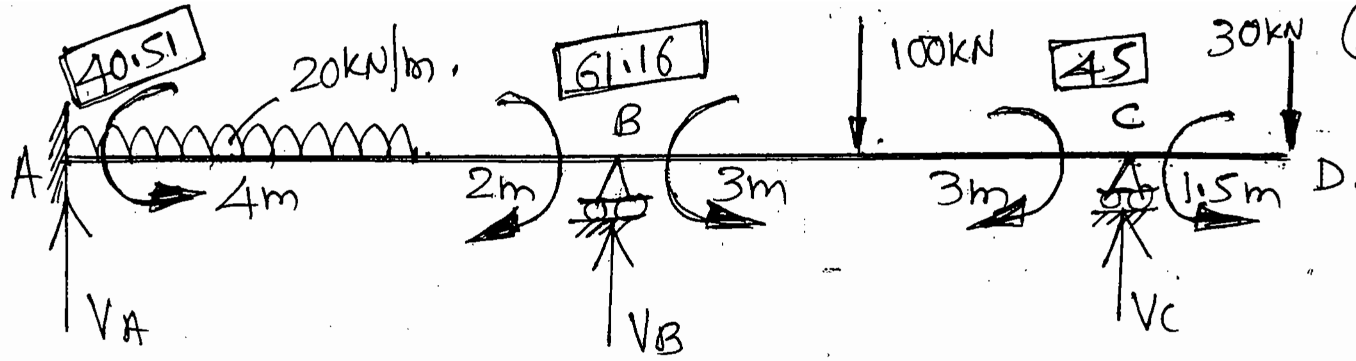
$\theta_C = -24.61/EI$

(d) <u>Final Moment</u>

$M_{AB} = -40.51$ kN-m ↻ $\qquad M_{BC} = -61.18$ kN-m ↻

$M_{BA} = 61.16$ kN-m ↺ $\qquad M_{CB} = 45$ kN-m ↺

$M_{CD} = -45$ kN-m ↻



$$\Sigma V = 0, \quad 20 \times 4 + 100 + 30 = V_A + V_B + V_C$$

$$\therefore \quad V_A + V_B + V_C = 210 \longrightarrow (i)$$

$\Sigma M_B = 0$, (LHS)

$$V_A \times 6 - 20 \times 4 \times \left(\tfrac{4}{2} + 2\right) - 40.51 + 61.16 = 0$$

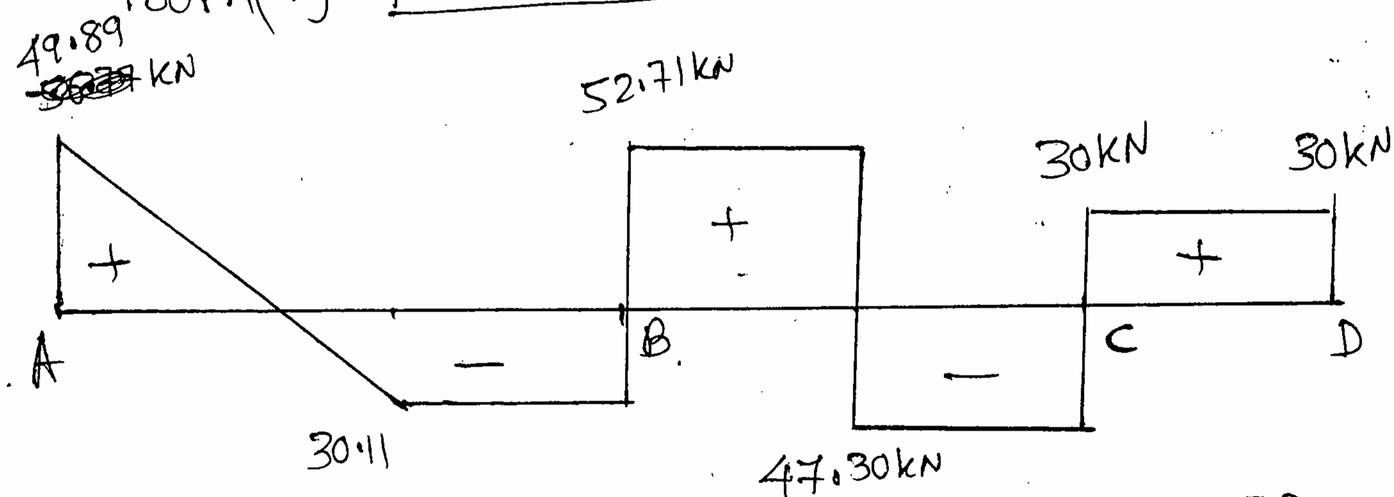
$$\boxed{V_A = \cancel{56.77}\ 49.89}\ \text{kN}$$

$\Sigma M_B = 0$ (RHS)

$$-V_C \times 6 + 100 \times 3 + 30 \times 7.5 - 61.16 + 45 - 45 = 0$$

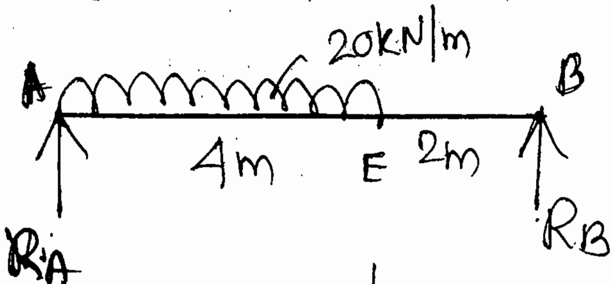
$$\boxed{V_C = 77.30\ \text{kN}}$$

From (i) $\boxed{V_B = \cancel{78.93}}$ 82.81 kN



SFD

Free BMD for AB

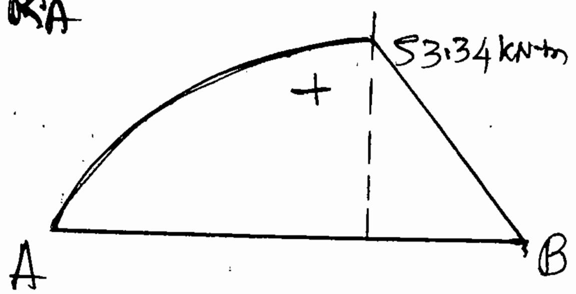


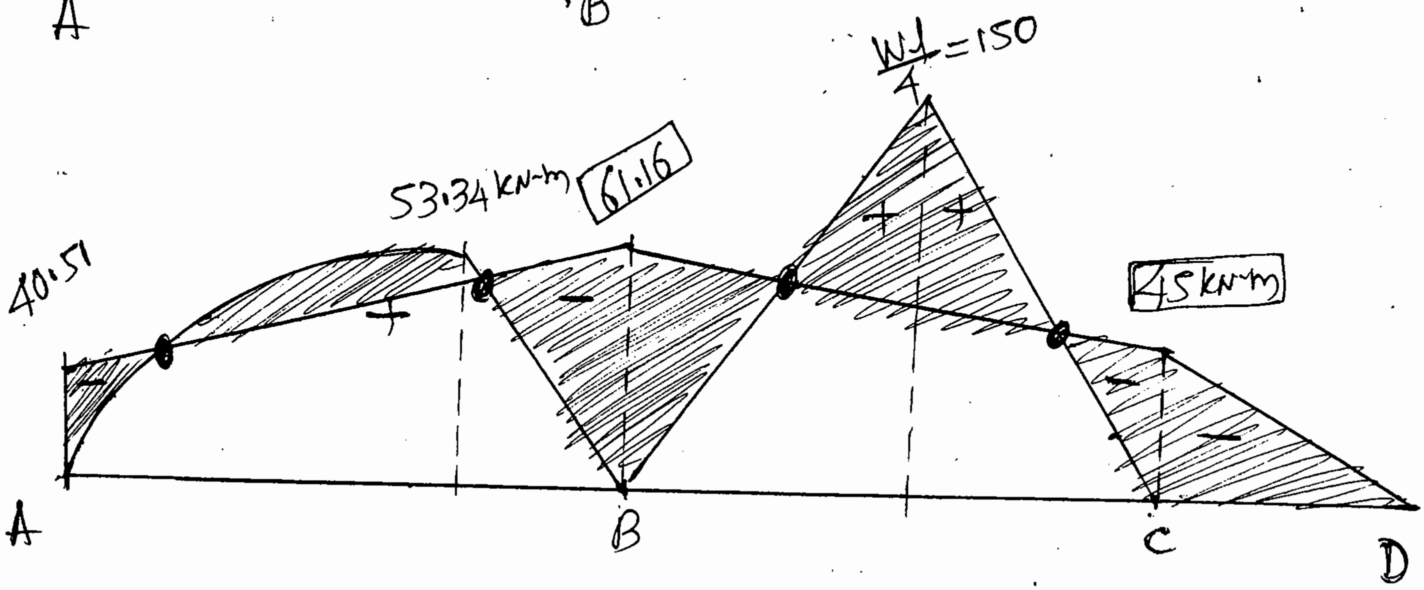
$R_A = 53.33\,kN$

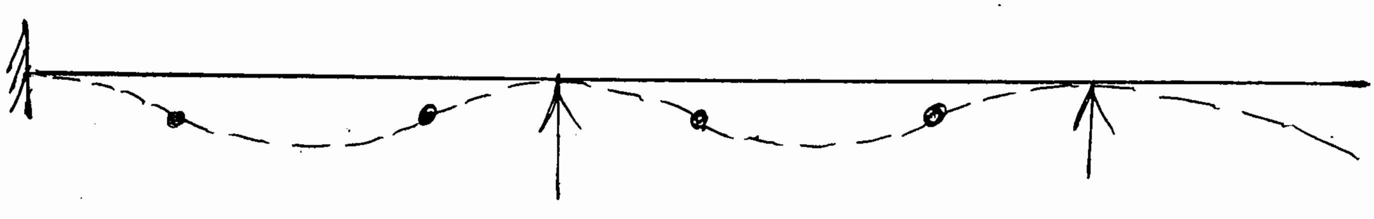
$R_B = 26.67\,kN$

$\therefore M_E = R_B \times 2$

$= \underline{\underline{53.34\,kN\text{-}m}}$



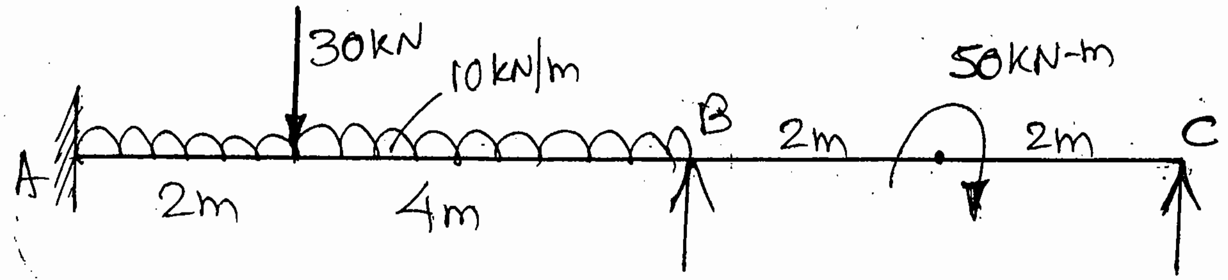




— × —

Sinking of Support

Eg:- 4] Analyse the beam shown by SD method and draw BMD, SFD & EC. The support 'B' sinks by 5mm. Take $E = 210 GPa$, $I = 0.1 G mm^4$



Sol^n

$$E = 210 GPa = 210 \times 10^9 \times 10^{-6} = 210 \times 10^3 \, N/mm^2$$

$$I = 0.1 G mm^4 = 0.1 \times 10^9 mm^4$$

$$EI = \left(\underset{N/mm^2}{210 \times 10^3}\right)\left(\underset{mm^4}{0.1 \times 10^9}\right) = 2.1 \times 10^{13} \, N\text{-}mm^2$$

$$EI = \frac{2.1 \times 10^{13}}{(1000)(1000)^2} = 2.1 \times 10^4 \, kN\text{-}m^2$$

(a) FEM

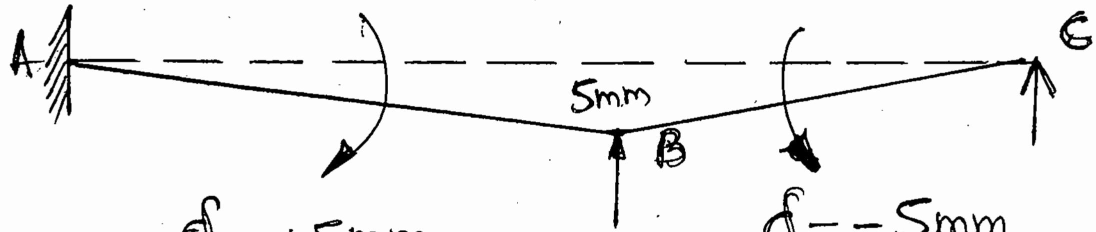
$$M_{FAB} = -\frac{wl^2}{12} - \frac{Wab^2}{l^2} = \frac{-10 \times 6^2}{12} - \frac{30 \times 2 \times 4^2}{6^2} = -56.67$$

$$M_{FBA} = +\frac{wl^2}{12} + \frac{Wa^2b}{l^2} = +\frac{10 \times 6^2}{12} + \frac{30 \times 2^2 \times 4}{6^2} = +43.33$$

$$M_{FBC} = M_{FCB} = +\frac{M}{4} = +12.5 \, kN\text{-}m$$

(b) SD Equation :

$\theta_A = 0$,



$\delta = +5mm$
$= +0.005m$

$\delta = -5mm$
$= -0.005m$

$$M_{AB} = \frac{2EI}{6}\left[0 + \theta_B - \frac{3 \times 0.005}{6}\right] - 56.67$$

$$= \frac{2(2.1 \times 10^4)}{6}\left[\theta_B - 2.5 \times 10^{-3}\right] - 56.67$$

$$= 7000\,\theta_B - 74.17 \longrightarrow \text{(i)}$$

$$M_{BA} = \frac{2(2.1 \times 10^4)}{6}\left[2\theta_B + 0 - 2.5 \times 10^{-3}\right] + 43.33$$

$$= 14000\,\theta_B + 25.83 \longrightarrow \text{(ii)}$$

$$M_{BC} = \frac{2(2.1 \times 10^4)}{4}\left[2\theta_B + \theta_C - \frac{3(-0.005)}{4}\right] + 12.5$$

$$= 21000\,\theta_B + 10500\,\theta_C + 51.87 \longrightarrow \text{(iii)}$$

$$M_{CB} = \frac{2(2.1 \times 10^4)}{4}\left[2\theta_C + \theta_B - \frac{3(-0.005)}{4}\right] + 12.5$$

$$= 21000\,\theta_C + 10500\,\theta_B + 51.87 \longrightarrow \text{(IV)}$$

(c) Equillibrium Condition

① At "B" $\boxed{M_{BA} + M_{BC} = 0}$

$$35000\,\theta_B + 10500\,\theta_C = -77.7 \longrightarrow (\mathrm{I})$$

② At 'C' $\boxed{M_{CB} = 0}$

$$10500\,\theta_B + 21000\,\theta_C = -51.87 \longrightarrow (\mathrm{II})$$

Solving

$$\theta_B = -1.74 \times 10^{-3}$$

$$\theta_C = -1.6 \times 10^{-3}$$

(d) Final Moment :

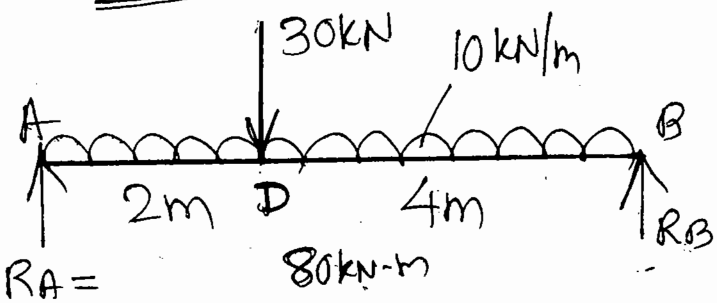
$M_{AB} = -86.35$ kN-m

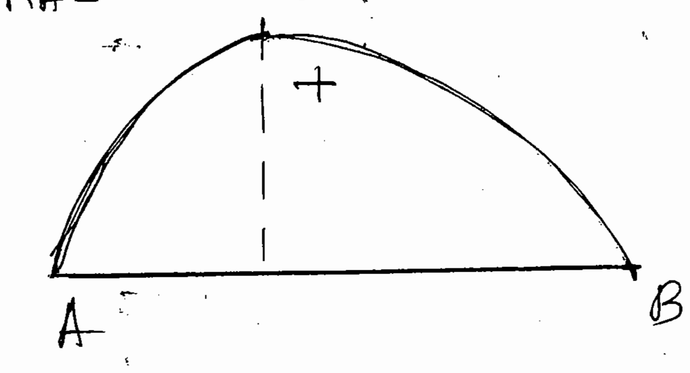
$M_{BA} = 1.47$ kN-m

$M_{BC} = -1.47$ kN-m

$M_{CB} = 0$

Free BMD for "AB"



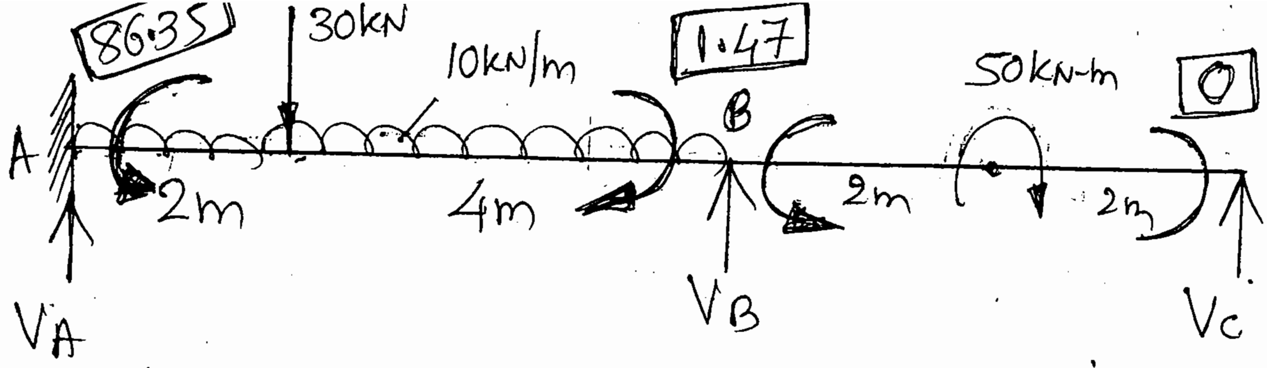


$R_A = 50$ kN

$R_B = 40$ kN

$$\therefore M_D = R_B \times 4 - 10 \times 4 \times \frac{4}{2}$$

$$= 80\ \text{kN-m}$$



$$V_A + V_B + V_C = 30 + 10 \times 6 = 90 \rightarrow (i)$$

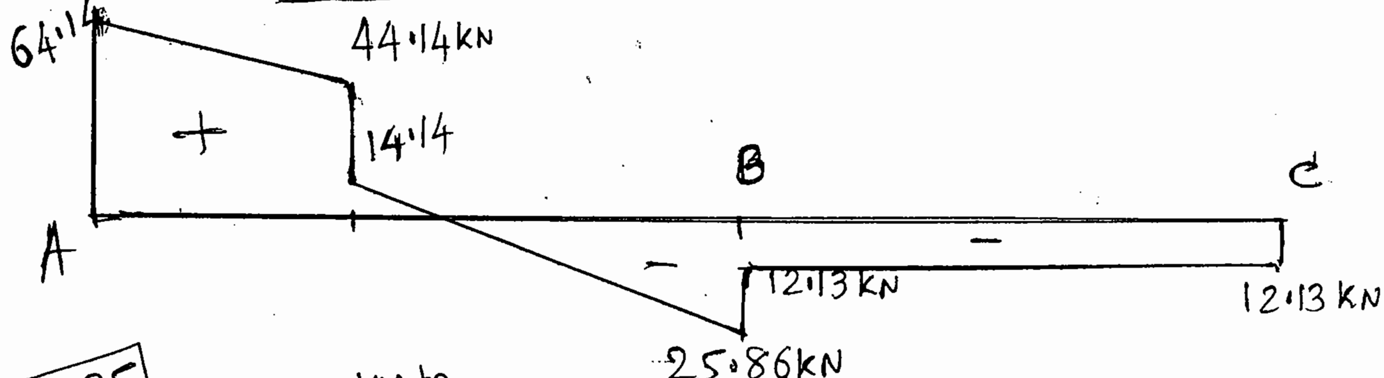
$\Sigma M_B = 0, (LHS)$

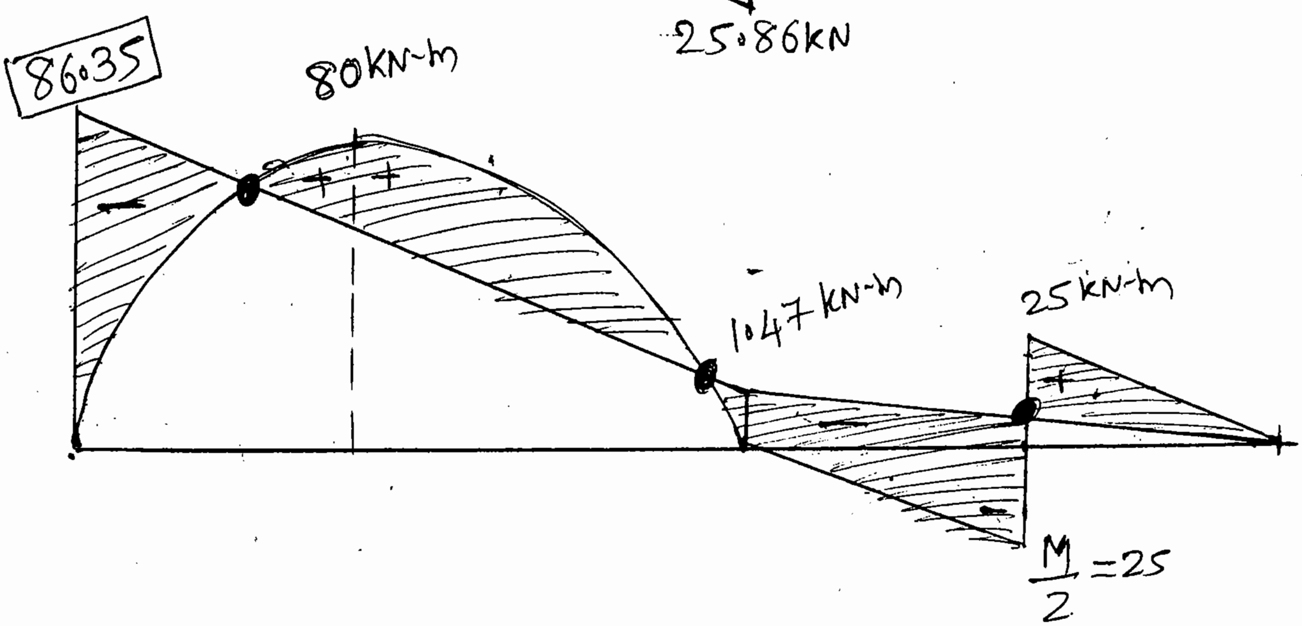
$$V_A \times 6 - 30 \times 4 - 10 \times 6 \times 6/2 - 86.35 + 1.47 = 0$$

$$\boxed{V_A = 64.14}$$

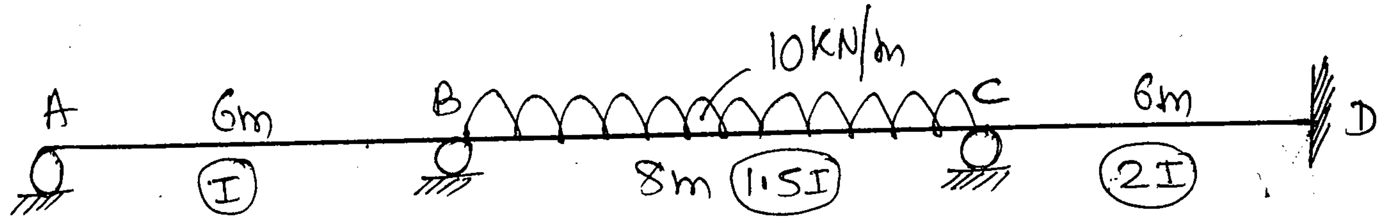
$\Sigma M_B = 0\ (RHS)$ $\quad -V_C \times 4 + 50 - 1.47 = 0 \quad \boxed{V_C = 12.13}$

From (i) $\boxed{V_B = 13.72}$



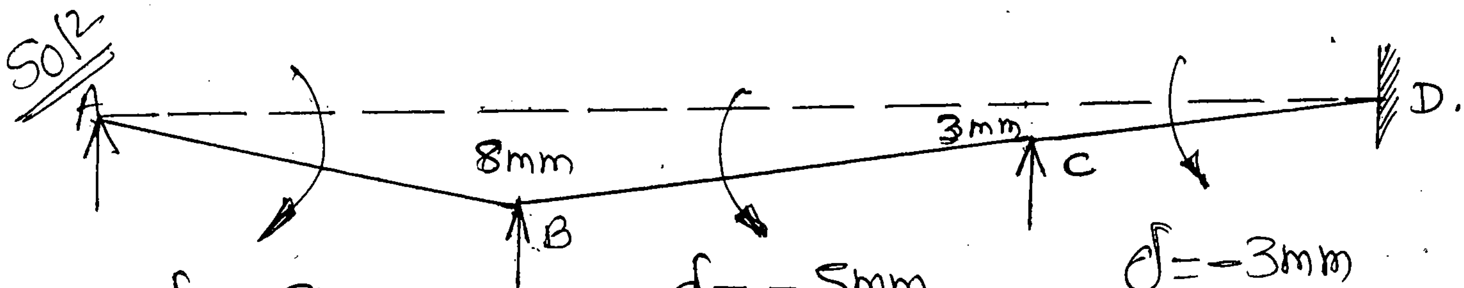


Eg:- 5] Analysis the beam shown by SD method. Draw BMD, SFD & EC.



Support B & C settles by 8mm and 3mm respt. Take $EI = 2 \times 10^4$ KN-m^2.

Sol:



$\delta = +8mm$
$0.008m$.

$\delta = -5mm$
$= -0.005m$.

$\delta = -3mm$
$= -0.003m$

(a) FEM

$$M_{FBC} = \frac{-wl^2}{12} = -53.33, \quad M_{FCB} = +53.33$$

(b) SD Equation :

$$\theta_D = 0$$

$$M_{AB} = \frac{2(2 \times 10^4)}{6}\left[2\theta_A + \theta_B - \frac{3(0.008)}{6}\right] + 0$$

$$= 13333.33\,\theta_A + 6666.67\,\theta_B - 26.67 \rightarrow (i)$$

$$M_{BA} = \frac{2(2\times10^4)}{6}\left[2\theta_B + \theta_A - \frac{3(0.008)}{6}\right] + 0$$

$$= 13333.33\,\theta_B + 6666.67\,\theta_A - 26.67 \quad \text{—(II)}$$

$$M_{BC} = \frac{2(1.5\times2\times10^4)}{8}\left[2\theta_B + \theta_C - \frac{3(-0.005)}{8}\right] - 53.33$$

$$= 15000\,\theta_B + 7500\,\theta_C - 39.26 \quad \text{—(III)}$$

$$M_{CB} = 7500\left[2\theta_C + \theta_B - \frac{3(-0.005)}{8}\right] + 53.33$$

$$= 15000\,\theta_C + 7500\,\theta_B + 67.39 \quad \text{—(IV)}$$

$$M_{CD} = \frac{2(2\times2\times10^4)}{6}\left[2\theta_C + 0 - \frac{3(-0.003)}{6}\right]$$

$$= 26666.67\,\theta_C + 20 \rightarrow \text{(V)}$$

$$M_{DC} = 13333.33\left(0 + \theta_C - \frac{3(-0.003)}{6}\right)$$

$$= 13333.33\,\theta_C + 20 \rightarrow \text{(VI)}$$

(c) Equillibrium Condition

(i) $\boxed{M_{AB} = 0}$

$$13333.33\,\theta_A + 6666.67\,\theta_B = 26.67 \rightarrow \text{(I)}$$

(ii) $\boxed{M_{BA} + M_{BC} = 0}$

$$28333.33\,\theta_B + 6666.67\,\theta_A + 7500\,\theta_C = 65.93 \rightarrow \text{(II)}$$

(iii) $\boxed{M_{CB} + M_{CD} = 0}$

$$7500\,\theta_B + 41666.67\,\theta_C = -87.39 \rightarrow \text{(III)}$$

Solving, $\theta_A = +5.56 \times 10^{-4}$ $\theta_C = -2.617 \times 10^{-3}$

$\theta_B = 2.889 \times 10^{-3}$

(d) Final Values

$M_{AB} \approx 0$

$M_{BA} = 15.55$ kN-m

$M_{BC} = -15.55$ kN-m

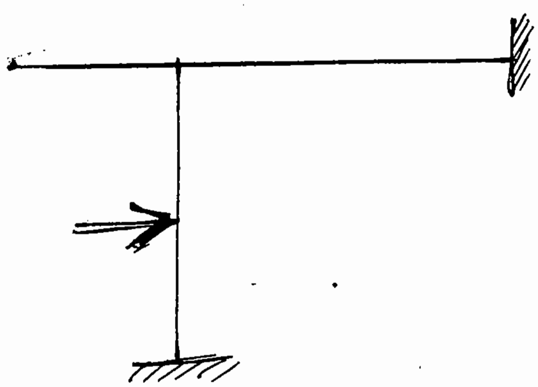
$M_{CB} = 49.80$ kN-m

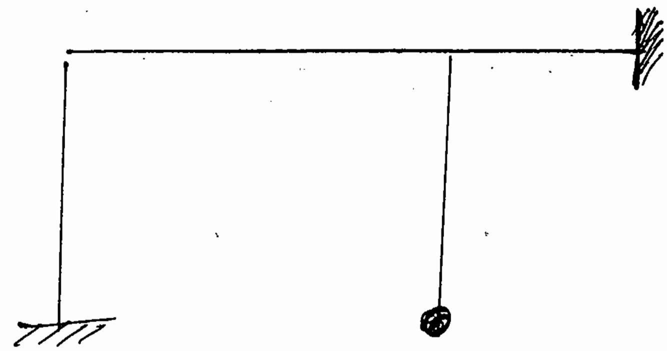
$M_{CD} = -49.80$ kN-m

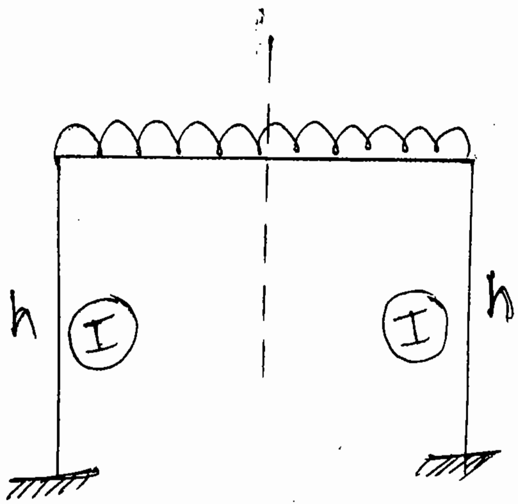
$M_{DC} = -14.89$ kN-m

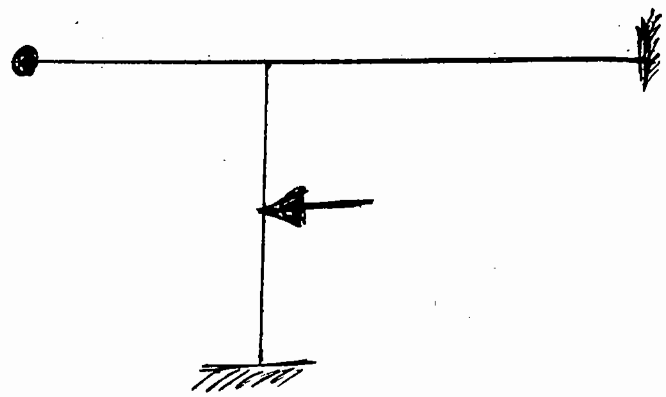
Date 14/9/18

Non Sway Frame :

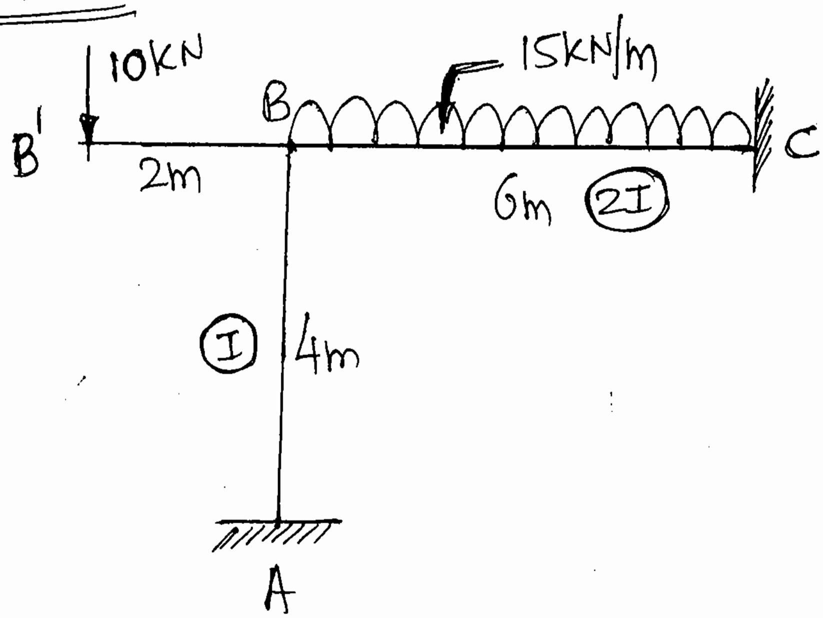








Eg:-1] Analyse the frame shown by S.D. method and draw BMD, SFD & EC.

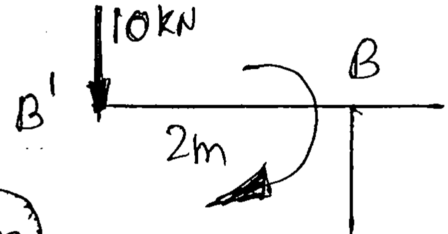


(a) FEM :

$M_{FAB} = M_{FBA} = 0$

$M_{FBC} = -\frac{\omega l^2}{12} = -\frac{15(6)^2}{12} = -45\,kN\text{-}m$

$M_{FCB} = +45.$



$M_{BB'} = +10 \times 2 = +20\,kN\text{-}m$

+ve sign for clockwise resisting moment.

(b) S.D. equation :-

$\theta_A = 0, \quad \theta_C = 0$ ($\because$ Fixed)

$\delta = 0$ ($\because$ Non-sway)

There is no equation for overhang BB'

$M_{AB} = \frac{2(EI)}{4}\left[\theta_B\right] = 0.5EI(\theta_B)$ —(i)

$M_{BA} = \frac{2EI}{4}\left[2\theta_B\right] = EI(\theta_B)$ —(ii)

$M_{BC} = \frac{2(2EI)}{6}\left[2\theta_B\right] - 45 = 1.33EI(\theta_B) - 45$ —(iii)

$M_{CB} = \frac{2(2EI)}{6}\left[\theta_B\right] + 45 = 0.666EI(\theta_B) + 45$ —(iv)

(c) Equillibrium Condition

At Intermediate joint "B"

$$\boxed{M_{BA} + M_{BC} + M_{BB'} = 0}$$

$$[EI\theta_B] + [1.33EI\theta_B - 45] + [20] = 0$$

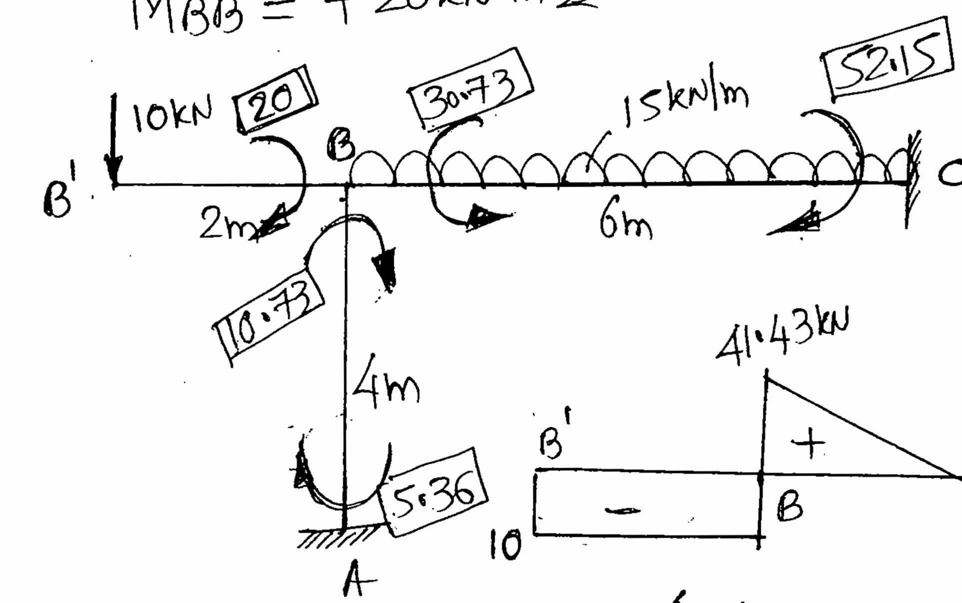
$$\therefore \theta_B = \frac{10.73}{EI}$$

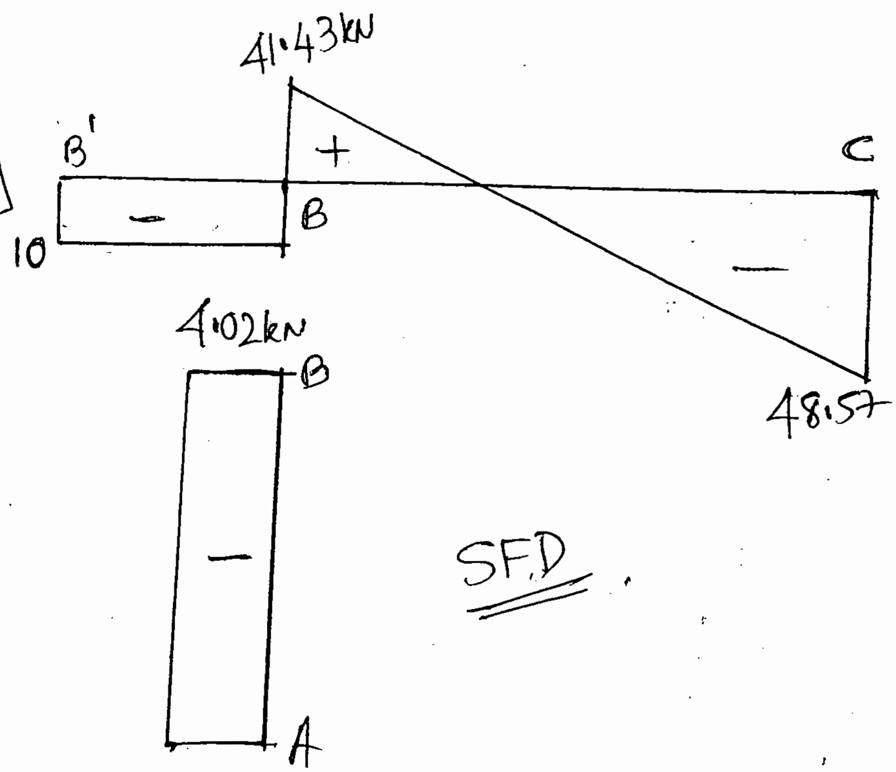
(d) Final Moment :- Substitute in eqn (i) to (iv)

$M_{AB} = 5.36$ kN-m $M_{BC} = -30.73$ kN-m

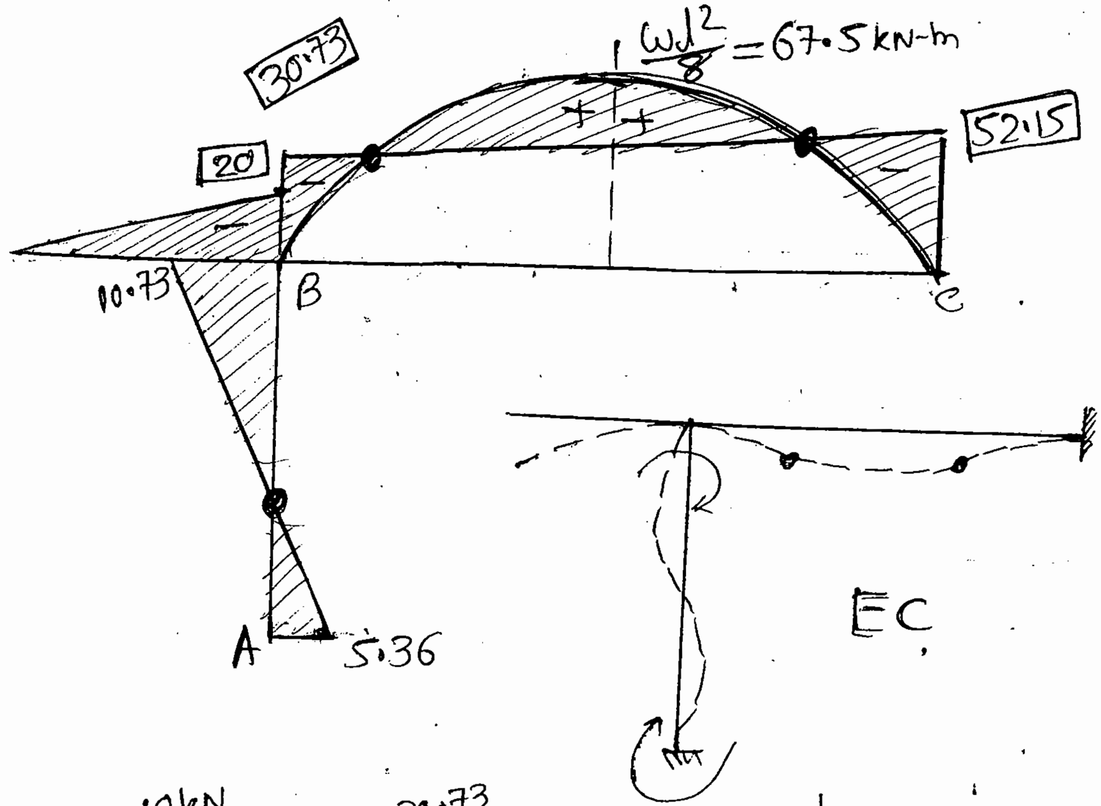
$M_{BA} = 10.73$ kN-m $M_{CB} = 52.15$ kN-m.

$M_{BB'} = +20$ kN-m

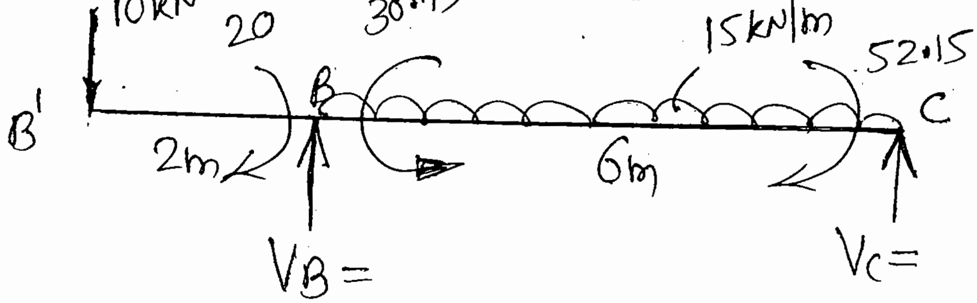




SFD



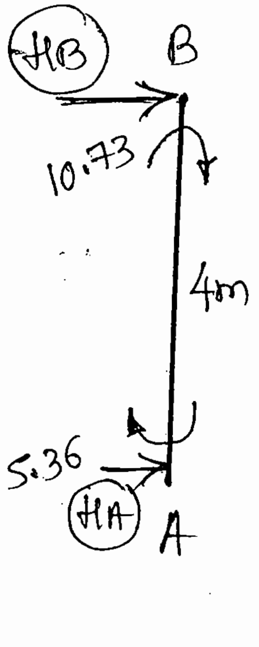
SFD



$$V_A + V_B = 10 + 15 \times 6 = 100 \quad \text{---(i)}$$

$$\Sigma M_C = 0, \quad -10 \times 8 + V_B \times 6 - 15 \times 6 \times 6/2 + 20 - 30.73 + 52.15 = 0$$

$$\boxed{V_B = 51.43} \quad \& \quad \boxed{V_C = 48.57}$$



$$\Sigma H = 0, \quad H_A + H_B = 0$$

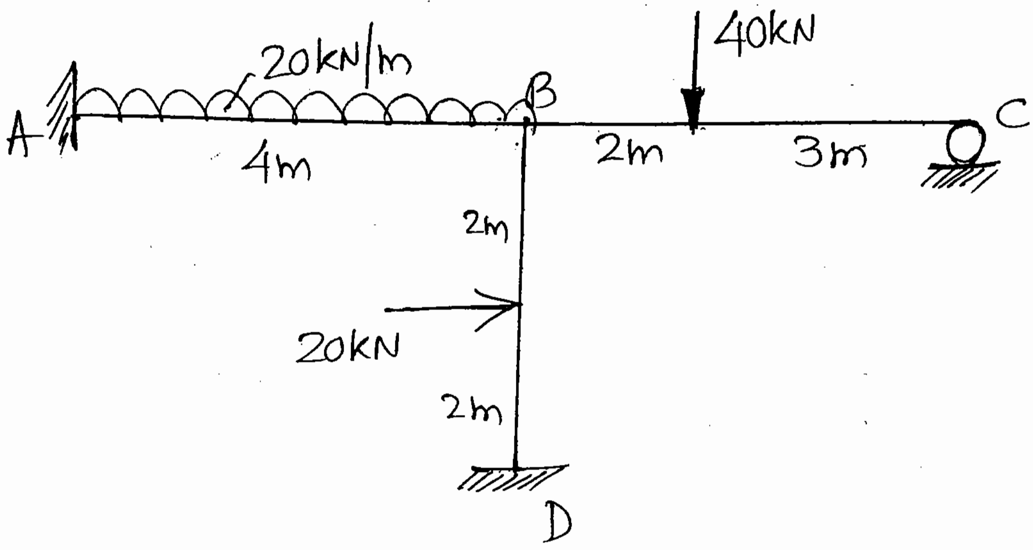
$$H_B \times 4 + 10.73 + 5.36 = 0$$

$$\boxed{H_B = -4.02}$$

$$\& \quad \boxed{H_A = +4.02}$$

—x—

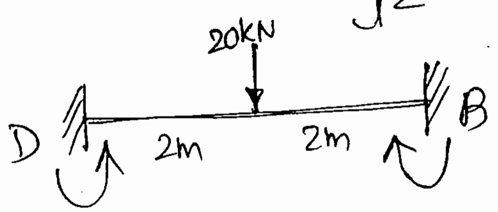
Eg:- 2] Analyse the frame shown by S.D. method. Draw BMD, SFD, EC



Sol: (a) FEM

$M_{FAB} = -\frac{\omega l^2}{12} = -26.67$, $M_{FBA} = +26.67$

$M_{FBC} = -\frac{Wab^2}{l^2} = -28.8$, $M_{FCB} = +\frac{Wa^2b}{l^2} = +19.2$



$M_{FDB} = -\frac{Wl}{8} = -10$

$M_{FBD} = +10$

(b) S.D. Equation :

$\theta_A = \theta_D = 0$ (Fixed)

$\delta = 0$ (Non-Sway)

$M_{AB} = \frac{2EI}{4}\left[\theta_B\right] - 26.67 = 0.5EI\theta_B - 26.67$ —(i)

$M_{BA} = \frac{2EI}{4}\left[2\theta_B\right] + 26.67 = EI\theta_B + 26.67$ —(ii)

$$M_{BC} = \frac{2EI}{5}\left[2\theta_B + \theta_C\right] - 28.8$$

$$= 0.8EI(\theta_B) + 0.4EI\theta_C - 28.8 \quad \text{—(III)}$$

$$M_{CB} = \frac{2EI}{5}\left[2\theta_C + \theta_B\right] + 19.2$$

$$= 0.8EI\theta_C + 0.4EI\theta_B + 19.2 \quad \text{—(IV)}$$

$$M_{BD} = \frac{2EI}{4}\left[2\theta_B\right] + 10 = EI(\theta_B) + 10 \quad \text{—(V)}$$

$$M_{DB} = \frac{2EI}{4}\left[\theta_B\right] - 10 = 0.5EI(\theta_B) - 10 \quad \text{—(VI)}$$

(c) Equillibrium Condition :—

at "B" $\boxed{M_{BA} + M_{BC} + M_{BD} = 0}$

$$2.8EI(\theta_B) + 0.4EI(\theta_C) = -7.87 \longrightarrow \text{I}$$

At "C" $\boxed{M_{CB} = 0}$

$$0.4EI(\theta_B) + 0.8EI(\theta_C) = -19.2 \longrightarrow \text{II}$$

solving

$$\theta_B = \frac{0.67}{EI}$$

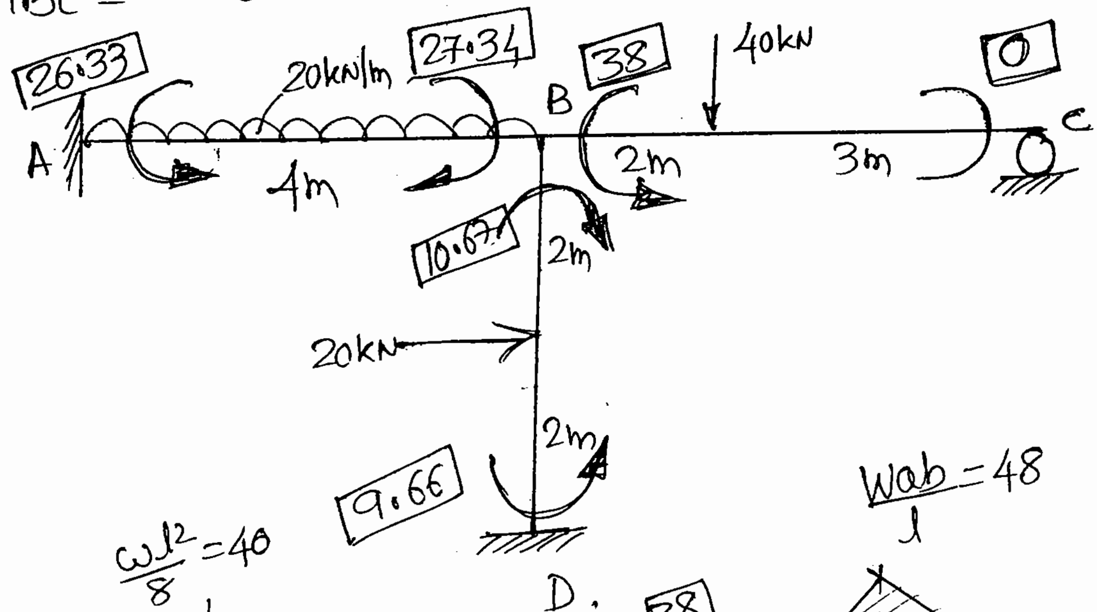
$$\theta_C = -\frac{24.33}{EI}$$

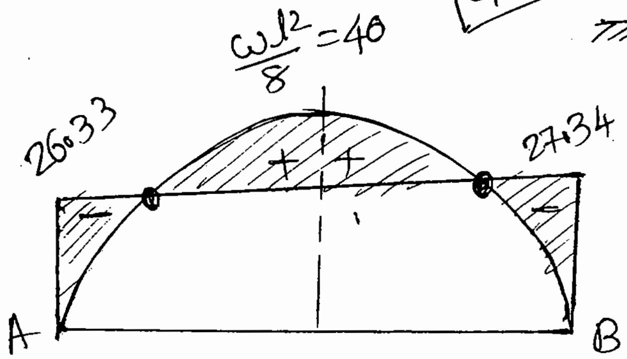
(d) Final Moment

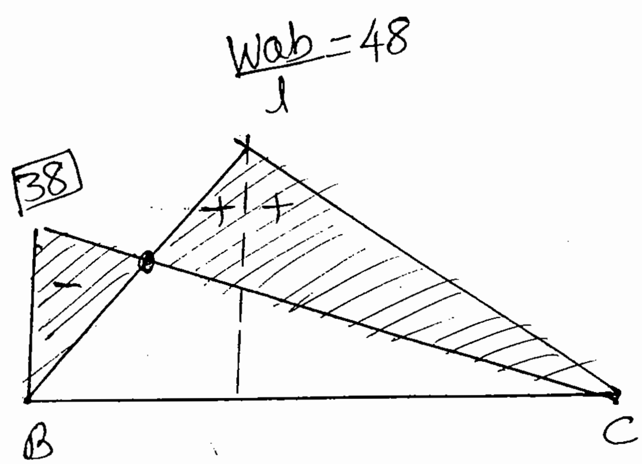
$M_{AB} = -26.33 \text{ kN-m}$ ↻ $\quad M_{CB} = 0$

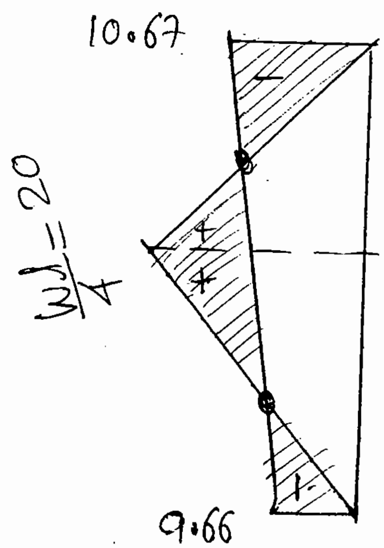
$M_{BA} = 27.34 \text{ kN-m}$ ↺ $\quad M_{BD} = 10.67 \text{ kN-m}$ ↺

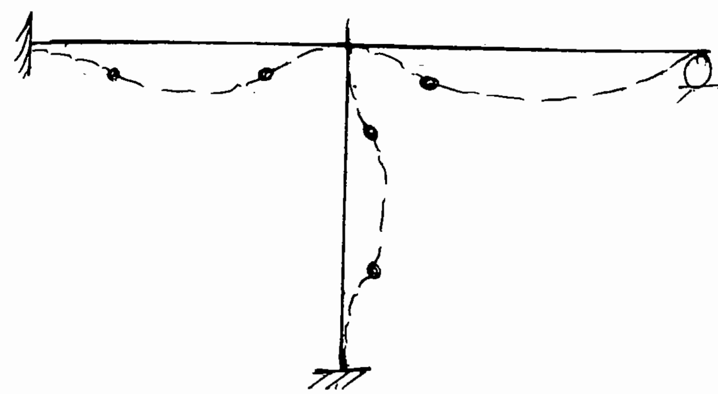
$M_{BC} = -38.00 \text{ kN-m}$ ↻ $\quad M_{DB} = -9.66 \text{ kN-m}$ ↻











Sunil. M. G

MODULE-2

Moment Distribution Method

(*) Procedure : 1> FEM

2> Distribution factor (DF)

3> Moment Distribution Table

4> Diagrams (SFD, BMD & EC)

1> FEM :

Refer Unit-2 — Slope Deflection method.

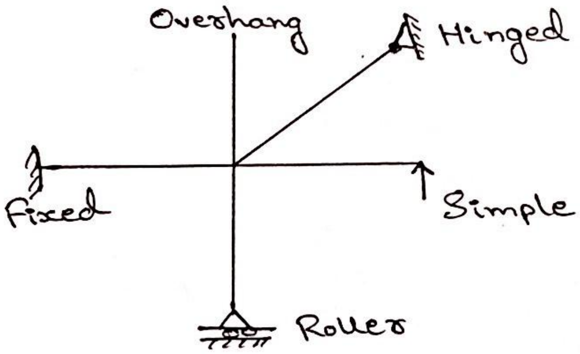
2> Distribution factor (DF) :

(For intermediate Support joints)

⇒

Joint	Member	Relative Stiffness(K)	ΣK	$DF = \frac{K}{\Sigma K}$

⇒ Relative Stiffness (K)



a) For Fixed (or) Continuous Support → $\left(K = \frac{I}{l}\right)$

b) For Simple, Roller (or) Hinge → $\left(K = \frac{3}{4} \times \frac{I}{l}\right)$

c) For Overhang → $(K = 0)$

3) Moment Distribution Table:

⇒ (a) If the far end is fixed (or) Continuous Carry 50% of moment with same sign.

(b) If the far end is "not Continuous", then there is no transfer of moments.

⇒ M.D. Table:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF						
FEM						
Bal C.O						
Bal C.O						
Bal C.O						
Bal C.O						
Final Moments						

4) Diagrams (SFD, BMD & EC):

Refer Unit-2 —— S.D. Notes

(*) Sinking and Rotation of Support

① FEM:

(a) Additional Moment due to rotation

@ Near end $= \frac{4EI\theta}{l}$

@ Far end $= \frac{2EI\theta}{l}$

(b) Additional Moment due to Sinking.

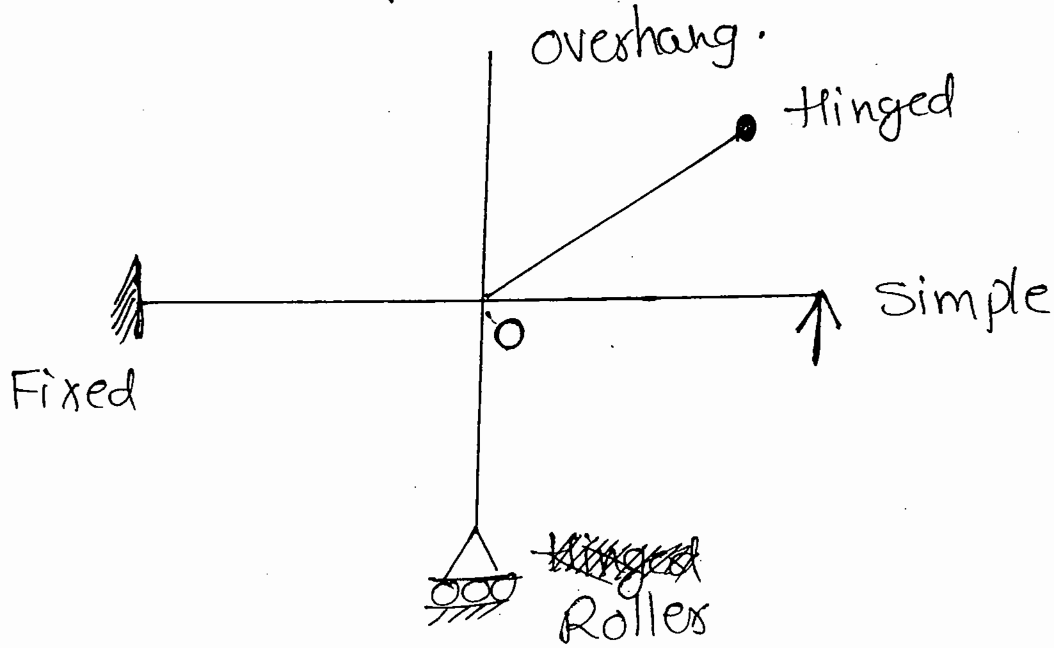
"$-\frac{6EI\delta}{l^2}$"

Date 21/9/18

(III) Moment Distribution Method

Relative stiffness $= \boxed{k = \frac{I}{l}}$

"The ratio of M.I to the span of beam is called relative stiffness."

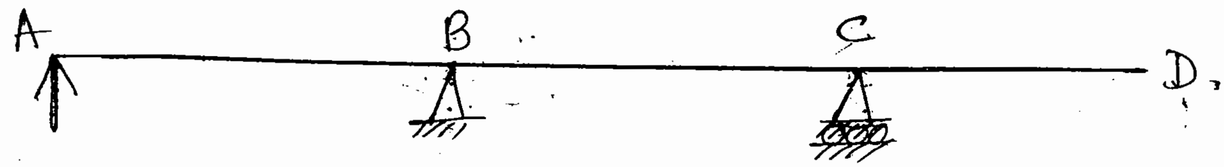


(a) For "Fixed end" or "Continuous" support $\Big\}$ $\boxed{k = \frac{I}{l}}$

(b) For 'Simple' or 'Hinge' or 'Roller' support $\Big\}$ $\boxed{k = \frac{3}{4}\frac{I}{l}}$

(c) For "Over hang" $\rightarrow$ $\boxed{k = 0}$

Continuous support :



(i) w.r.to "B" ⟶ 'A' is Not continuous $K = \frac{3}{4}\frac{I}{l}$

'C' is Not Continuous $K = \frac{3}{4}\frac{I}{l}$.

ii) w.r.to 'C' ⟶ 'B' is Continuous $k = I/l$

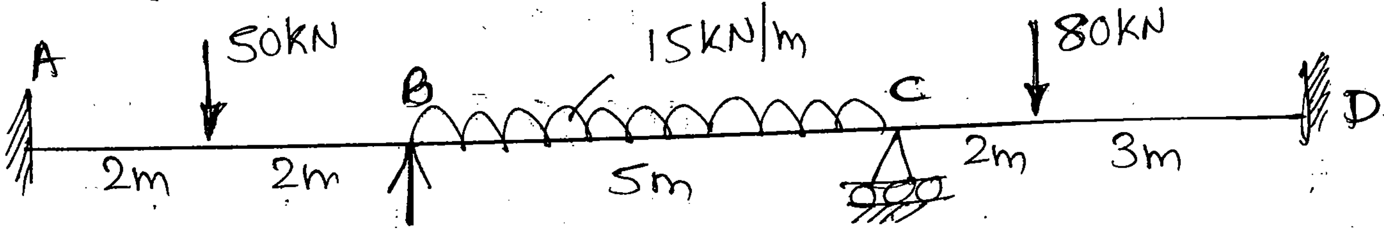
'D' is overhang $k = 0$

Carry over of Moments:-

(i) If the far end is "fixed" or "Continuous" 50% of moment with take or carry Same sign.

(ii) If far end is Not continuous, then there is no Transfer of moment.

Eg:- 1 Analyse the continuous beam shown by MD method. Draw SFD, BMD & EC.



Sol^{n} (a) FEM

$M_{FAB} = -\frac{Wl}{8} = -25$ kN-m, $M_{FBA} = +25$ kN-m

$M_{FBC} = -\frac{\omega l^2}{12} = -31.25$, $M_{FCB} = +\frac{\omega l^2}{12} = 31.25$

$M_{FCD} = -\frac{Wab^2}{l^2} = -57.6$, $M_{FDC} = +\frac{Wa^2b}{l^2} = 38.4$

(b) Distribution Factor (For Intermediate support)

Joint	Member	Relative Stiffness = K	Sum ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$(I/l) = I/4 = 0.25I$	$0.45I$	$\frac{0.25}{0.45} = 0.56$
	BC	$(I/l) = I/5 = 0.20I$		$\frac{0.2}{0.45} = 0.44$
C	CB	$(I/l) = I/5 = 0.20I$	$0.4I$	0.5
	CD	$(I/l) = I/5 = 0.2I$		0.5

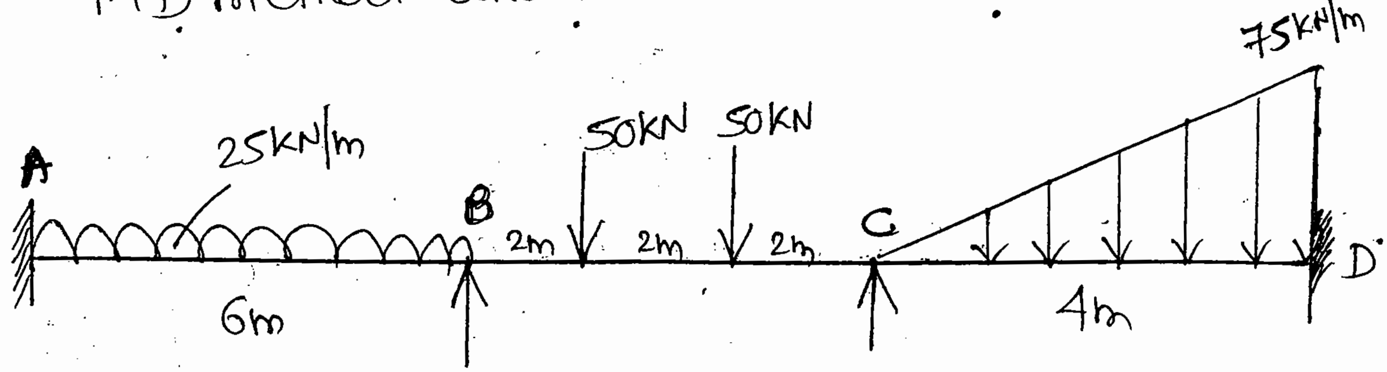
(c) Moment Distribution Table

A (Fixed) B C D (Fixed)

AB	BA	BC	CB	CD	DC	Member
-	0.56	0.44	0.5	0.5		DF
-25	25	-31.25	31.25	-57.6	38.4	FEM
	3.5	2.75	13.18	13.18		Balance
1.75		6.59	1.37		6.59	Carry over
	-3.69	-2.90	-0.68	-0.68		Bal
-1.84		-0.34	-1.45		-0.34	C.O
	0.19	0.15	0.73	0.73		Bal
0.09		0.36	0.075		0.36	C.O
	-0.20	-0.16	-0.037	-0.037		Bal
-0.10		-0.018	-0.08		-0.018	C.O
	0.01	0.008	0.04	0.04		Bal
-25.10	24.81	-24.81	44.40	-44.40	44.992	Final Moments,

Draw SFD, BMD and EC.

Eg:- 2] Analyse the beam shown by MD method and draw BMD, SFD & EC.



Sol:

(a) FEM

$M_{FAB} = -\frac{\omega l^2}{12} = -75$, $M_{FBA} = +75$

$M_{FBC} = -\frac{Wab^2}{l^2} = -\left[\frac{50\times2\times4^2}{6^2} + \frac{50\times4\times2^2}{6^2}\right] = -66.67$

$M_{FCB} = +\frac{Wa^2b}{l^2} = +66.67$

$M_{FCD} = -\frac{\omega l^2}{30} = \frac{-75\times4^2}{30} = -40$

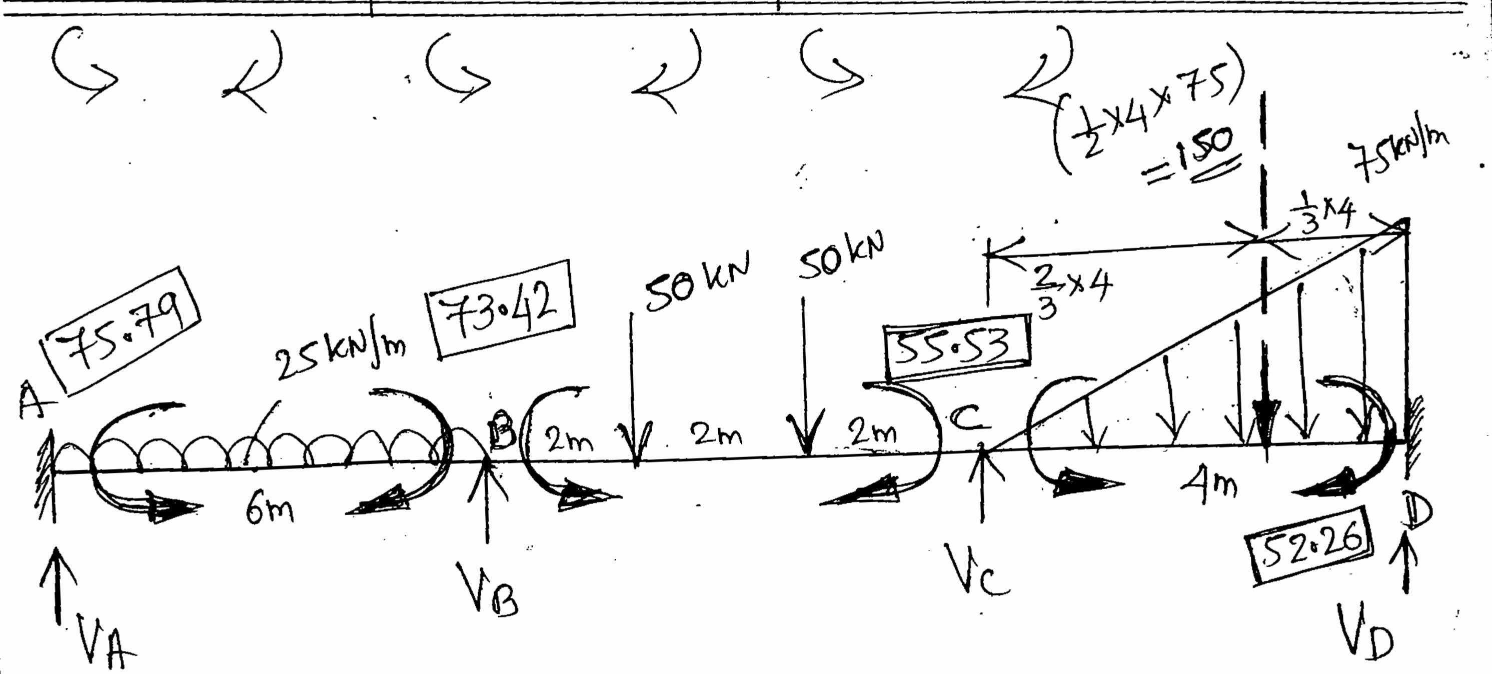
$M_{FDC} = +\frac{\omega l^2}{20} = +60$

(b) D.F : (For Intermediate Support)

	Member	Relative stiffness = K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$I/l = I/6 = 0.167I$	0.333I	0.5
	BC	$I/l = I/6 = 0.167I$		0.5
C	CB	$I/l = I/6 = 0.167I$	0.417I	0.4
	CD	$I/l = I/4 = 0.25I$		0.6

(C) M.D. Table

A (Fixed) AB	BA	B BC	CB	C CD	D (Fixed) DC	Member
	0.5	0.5	0.40	0.60		DF
-75	75	-66.67	+66.67	-40	60	FEM
	-4.17	-4.17	-10.67	-16.0		Balance
-2.09		-5.34	-2.09		-8	C.O
	2.67	2.67	0.84	1.25		Bal
1.34		0.42	1.34		0.63	C.O
	-0.21	-0.21	-0.54	-0.80		Bal
-0.11		-0.27	-0.11		-0.4	C.O
	0.14	0.14	+0.105	+0.06		Bal
0.07		0.02	0.07		0.03	C.O
	-0.01	-0.01	-0.03	-0.04		Bal
-75.79	73.42	-73.42	55.53	-55.53	52.26	Final Values,



Reactions

$\Sigma V = 0, \; V_A + V_B + V_C + V_D = 25\times 6 + 2\times 50 + 150$

$= 400 \quad \text{—(i)}$

$\Sigma M_B = 0$ (LHS)

$V_A \times 6 - 25\times 6\times 6/2 - 75.79 + 73.42 = 0 \quad \boxed{V_A = 75.4}$

$\Sigma M_C = 0$ (RHS)

$-V_D \times 4 + \left(150 \times \frac{2}{3} \times 4\right) - 55.53 + 52.26 = 0$

$\boxed{V_D = 99.18}$

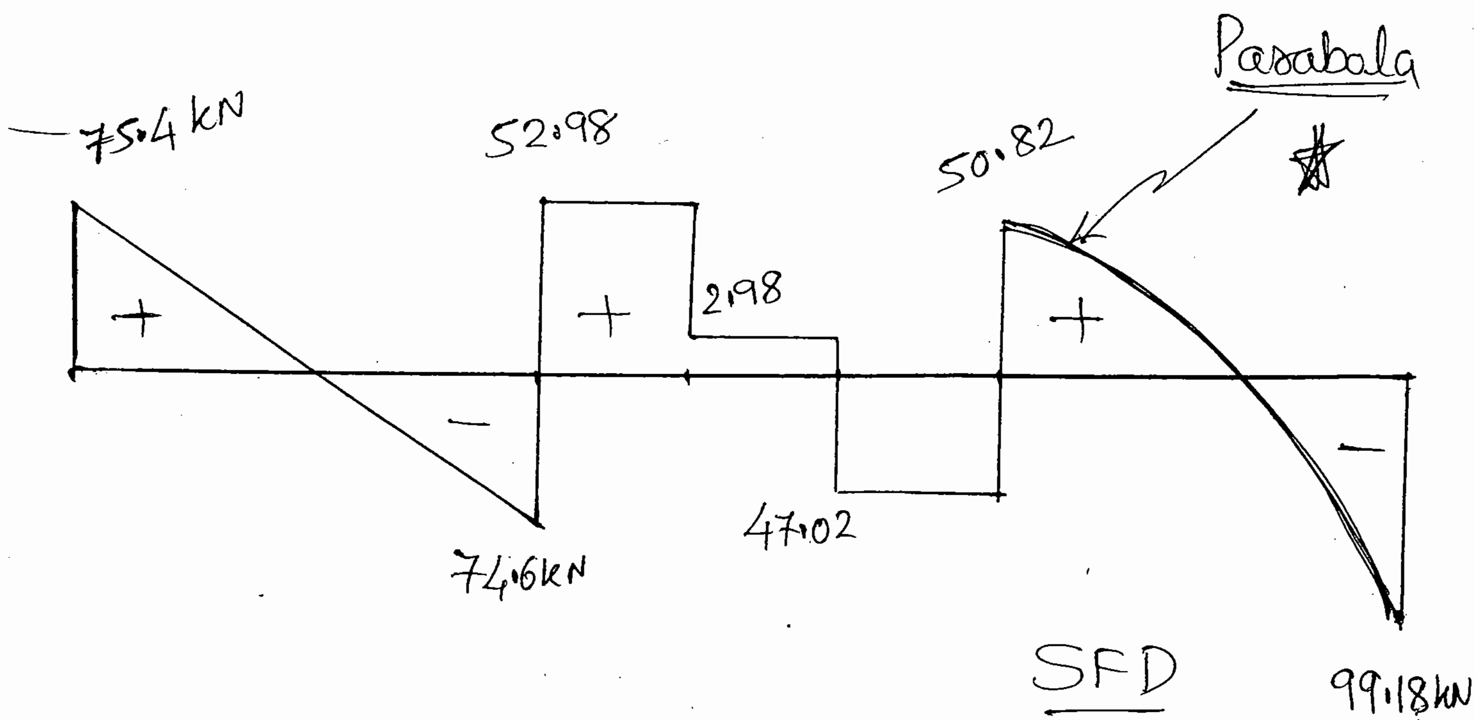
$\Sigma M_C = 0$ (LHS)

$75.4 \times 12 + V_B \times 6 - 25\times 6\times 9 - 50\times 2 - 50\times 4$

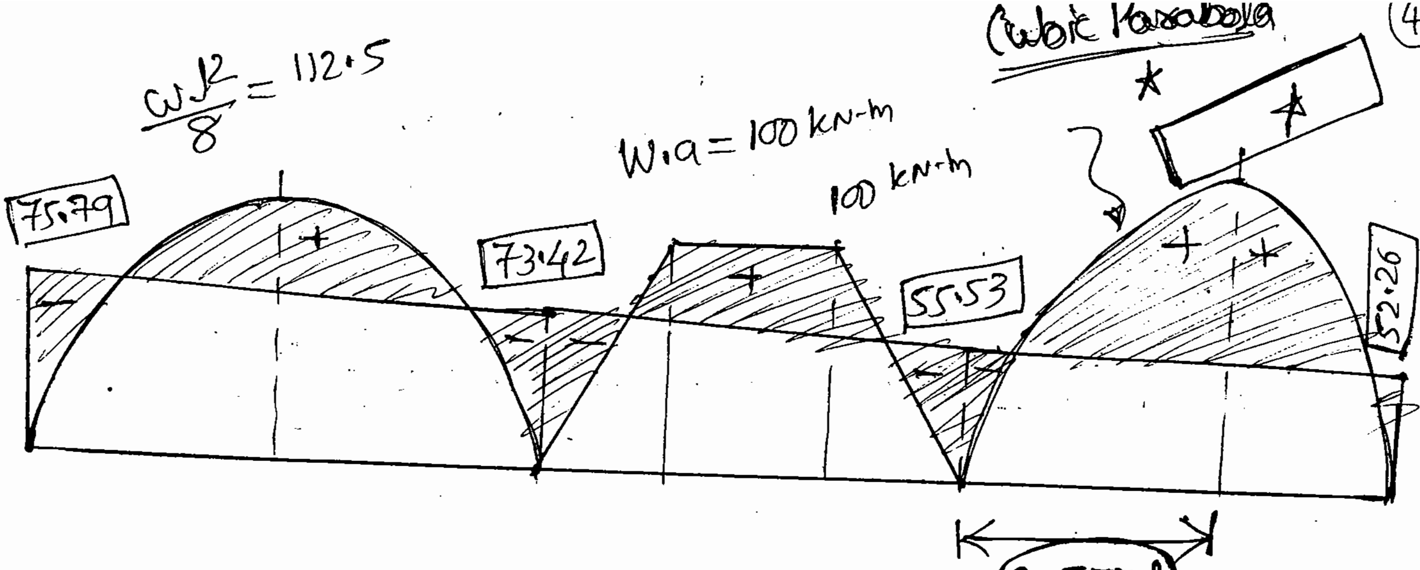
$- 75.79 + \cancel{73.42} - \cancel{73.42} + 55.53 = 0$

$\boxed{V_B = 127.58}$

From (i) $\boxed{V_C = 97.84}$



SFD



$0.577l$

$= 0.577 \times 4$

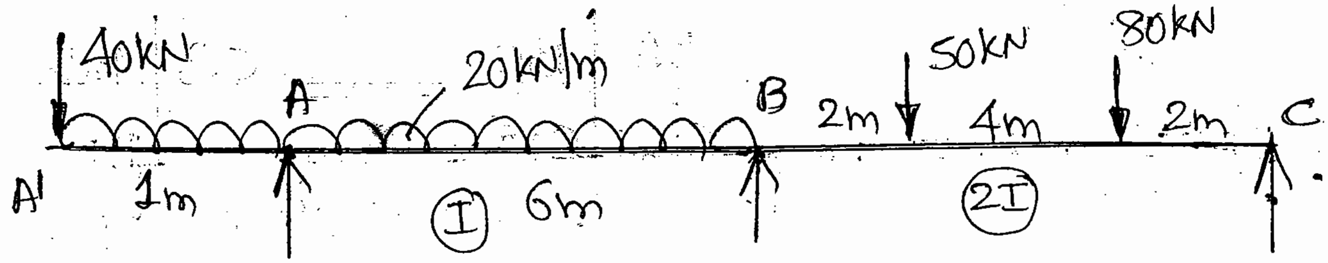
$= 2.308m$

$\boxed{M = 0.06415 \omega l^2}$ *

$= 0.06415 \times 75 \times (4)^2 = 77$ kN-m

—x—

Eg:- 3] Analyse the beam shown by M.D. method. Draw SFD, BMD & EC.

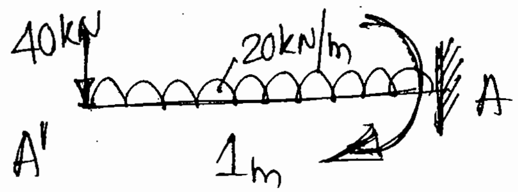


(a) FEM

$$M_{FAB} = -\frac{wl^2}{12} = -60 \text{ kN-m}, \quad M_{FBA} = +60 \text{ kN-m}$$

$$M_{FBC} = -\frac{Wab^2}{l^2} = -\left[\frac{50 \times 2 \times 6^2}{8^2} + \frac{80 \times 6 \times 2^2}{8^2}\right] = -86.25 \text{ kN-m}$$

$$M_{FCB} = +\frac{Wa^2b}{l^2} = +\left[\frac{50 \times 2^2 \times 6}{8^2} + \frac{80 \times 6^2 \times 2}{8^2}\right] = +108.75 \text{ kN-m}$$



$$M_{AA'} = +40 \times 1 + 20 \times 1 \times \tfrac{1}{2}$$
$$= +50 \text{ kN-m}.$$

(b) D.F. (For Intermediate)

	Member	K	ΣK	$DF = \frac{K}{\Sigma K}$
A	AA'	0 ($\because$ Overhang)	0.167I	0
	AB	$\left(\frac{I}{l}\right) = \frac{I}{6} = 0.167I$		1
B	BA	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{I}{6}\right) = 0.125I$	0.3125I	0.40
	BC	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{2I}{8}\right) = 0.1875I$		0.60

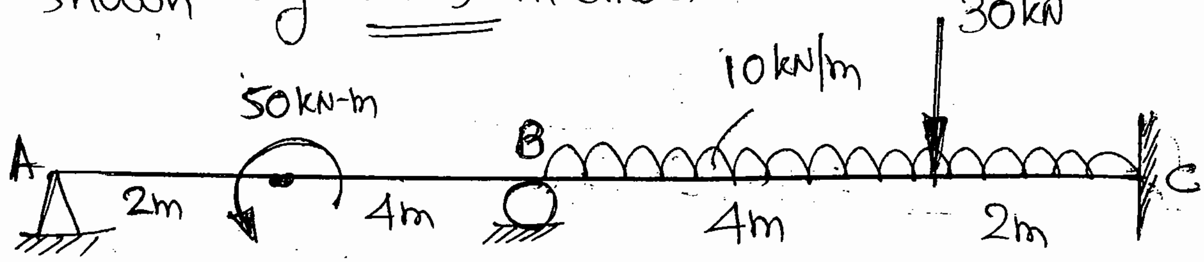
M.D. Table

A, B — C: Simple Hinge Roller

	A		B		C
AA'	AB	BA	BC	CB	Members
0	1	0.4	0.6		DF
50	-60	60	-86.25	108.75	FEM
				-108.75	Release C
			-54.37		C.O
50	-60	60	-140.62	0	Initial Values
0	10	32.25	48.37		Bal
	0	5		0	C.O
		-2	-3		Bal
	0			0	C.O
50	-50	95.25	-95.25	0	Final Values

Refer S.D. Notes for SFD, BMD.

Eg:-4] Analyse the continuous beam shown by M.D. method.



Sol^n (a) FEM

$$M_{FAB} = -\frac{Mb}{l^2}(2a-b) = \frac{-50\times4(2\times2-4)}{6^2} = 0$$

$$M_{FBA} = -\frac{Ma}{l^2}(2b-a) = -\frac{50\times2}{6^2}(2\times4-2) = -16.67 \text{ kN-m}$$

$$M_{FBC} = -\frac{wl^2}{12} - \frac{Wab^2}{l^2} = -43.33 \text{ kN-m}$$

$$M_{FCB} = +\frac{wl^2}{12} + \frac{Wa^2b}{l^2} = 56.67 \text{ kN-m.}$$

(b) D.F. (For Intermediate support)

		k	Σk	$DF = \frac{k}{\Sigma k}$
B	BA	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{I}{6}\right) = 0.125I$	0.292I	0.43
	BC	$\left(\frac{I}{l}\right) = \frac{I}{6} = 0.167I$		0.57

M.D. Table

A (Hinge) | B | C (Fixed)

AB	BA	BC	CB	Member
-	0.43	0.57		DF
0	-16.67	-43.33	56.67	FEM
0	0			Release (A) C.O.
0	-16.67	-43.33	56.67	Initial Values.
0	25.80	34.20	17.10	Bal C.O
0	9.13	-9.13	73.77	Final Values

Draw BMD, SFD.

Sinking and Rotation of Support

Additional Moment due to Rotation

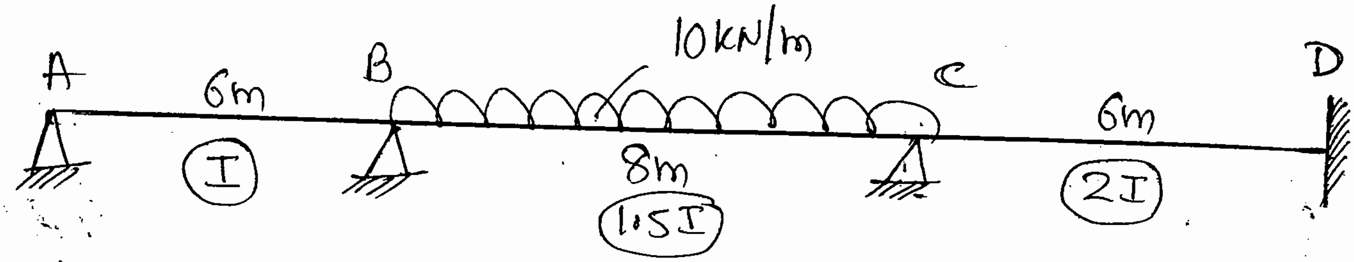
(i) Additional Moment at Near end $\Big\} = \dfrac{4EI\theta}{l}$

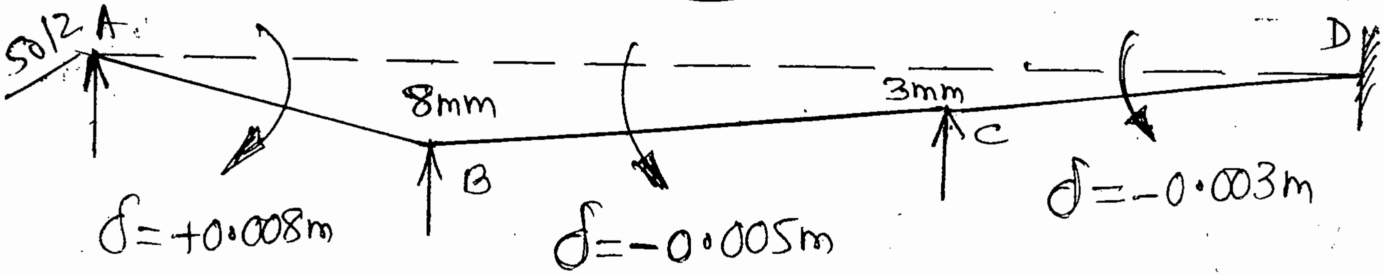
(ii) Additional moment at Far end $\Big\} = \dfrac{2EI\theta}{l}$.

(iii) Additional moment due to Sinking $\Big\} = -\dfrac{6EI\delta}{l^2}$

★ ∴ The above additional moments are added to F.E.M ★

Eg:– 5] Analyse the continuous beam shown by M.D. method and draw SFD, BMD. Support 'B' and 'C' settle by 8mm and 3mm respt. $EI = 2\times10^4\, kN/m^2$





(a) FEM

$$M_{FAB} = 0 - \frac{6EI\delta}{l^2} = 0 - \frac{6(1\times2\times10^4)(0.008)}{6^2} = -26.67$$

$$M_{FBA} = 0 - \frac{6EI\delta}{l^2} = 0 - \frac{6(1\times2\times10^4)(0.008)}{6^2} = -26.67$$

$$M_{FBC} = -\frac{wl^2}{12} - \frac{6EI\delta}{l^2} = \frac{-10\times8^2}{12} - \frac{6(1.5\times2\times10^4)(-0.005)}{8^2}$$

$$= -39.27\, kN\text{-}m$$

$$M_{FCB} = +\frac{wl^2}{12} - \frac{6EI\delta}{l^2} = \frac{10\times8^2}{12} - \frac{6(1.5\times2\times10^4)(-0.005)}{8^2}$$

$$= +67.40\, kN\text{-}m$$

$$M_{FCD} = 0 - \frac{6EI\delta}{l^2} = 0 - \frac{6(2\times 2\times 10^7)(-0.003)}{6^2}$$

$$= +20 \text{ kN-m}$$

$$M_{FDC} = 0 - \frac{6EI\delta}{l^2} = +20 \text{ kN-m}$$

(b) D.F

		K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\times\frac{I}{6} = 0.125I$	0.3125I	0.4
	BC	$I/l = \frac{1.5I}{8} = 0.1875I$		0.6
C	CB	$I/l = \frac{1.5I}{8} = 0.1875I$	0.5200I	0.36
	CD	$I/l = \frac{2I}{6} = 0.333I$		0.64

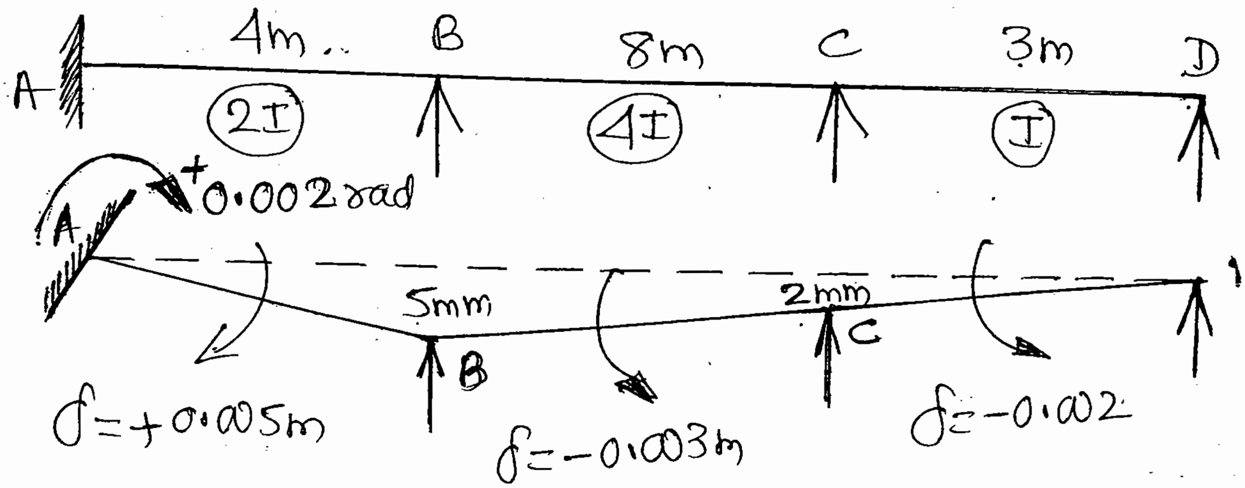
(c) M.D Table

Fixed

AB	BA	BC	CB	CD	DC	Member
	0.4	0.6	0.36	0.64		DF
−26.67	−26.67	−39.27	67.40	20	20	FEM
+26.67	13.33					Release (A) C.O
0	−13.33	−39.27	67.40	20	20	Initial
	21.04	31.56	−31.46	−55.94		Bal
0		−15.88	15.76		−27.97	C.O
	6.35	9.53	−5.67	−10.08		Bal
0		−2.83	4.76		−5.04	C.O
	1.13	1.70	−1.71	−3.04		Bal
0		−0.85	0.85		−1.52	C.O
	0.34	0.51	−0.30	−0.54		Bal
0		−0.15	0.25		−0.27	C.O
0	+0.06	0.09	−0.09	−0.16		Bal
	15.59	−15.59	49.79	−49.79	−14.80	Final

Eg:- 6 fig shows a continuous beam ABCD. Analyse the beam by M.D method. If the End "A" rotates by 0.002 radians in the clockwise order & Support 'B' sinks by 5mm & 'C' by 2mm. Take

$EI = 18000 \ kN\text{-}m^2$



(a) FEM

$$M_{FAB} = 0 + \underbrace{\frac{4EI\theta}{l}}_{\text{Near End}} - \underbrace{\frac{6EI\delta}{l^2}}_{\text{Sinking}}$$

$$= 0 + \frac{4(2\times18000)(0.002)}{4} - \frac{6(2\times18000)(0.005)}{4^2} = 4.5$$

$$M_{FBA} = 0 + \underbrace{\frac{2EI\theta}{l}}_{\text{Far end}} - \frac{6EI\delta}{l^2}$$

$$= 0 + \frac{2(2\times18000)(0.002)}{4} - \frac{6(2\times18000)(0.005)}{4^2} = -31.5$$

$$M_{FBC} = 0 - \frac{6EI\delta}{l^2} = 0 - \frac{6(4\times18000)(-0.003)}{8^2} = 20.25$$

$$M_{FCB} = 0 - \frac{6EI\delta}{l^2} = 20.25$$

$$M_{FCD} = 0 - \frac{6EI\delta}{l^2} = -\frac{6(1\times18000)(-0.002)}{3^2}$$

$$= 24 \text{ kN-m}$$

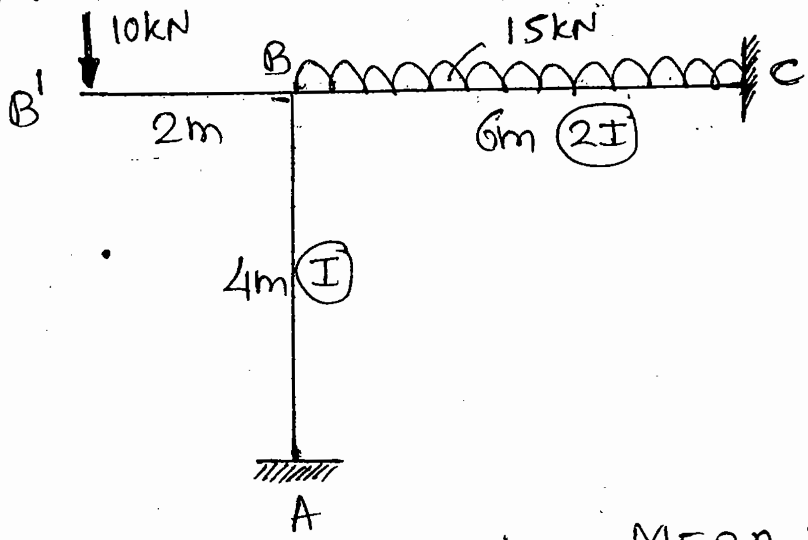
$$M_{FDC} = 0 - \frac{6EI\delta}{l^2} = 24 \text{ kN-m}$$

(b)

Date 05/10/8

: Non-Sway Frames :-

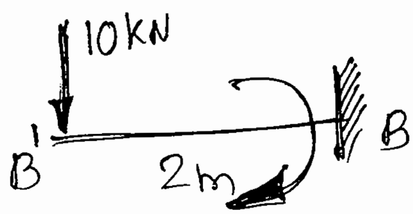
Eg:- 1] Analyse the rigid frame by M.D. method. Draw SFD, BMD & EC.



(a) FEM:

$$M_{FAB} = M_{FBA} = 0$$

$$M_{FBC} = \frac{-wl^2}{12} = -45, \quad M_{FCB} = +45 \text{ kN-m}$$



$$M_{BB'} = +10 \times 2 = +20 \text{ kN-m}$$

(clockwise resisting moment).

(b) D.F (For Intermediate)

		K	ΣK	$DF = \frac{K}{\Sigma K}$
	BA	$I/l = I/4 = 0.25I$		0.43
B	BC	$I/l = \frac{2I}{6} = 0.33I$	0.58I	0.57
	BB'	0		0

(C) M.D. Table

BB'	AB	BA	BC	CB	Member
0		0.43	0.57		DF
20	0	0	-45	45	FEM
0		10.75	14.25		Bal
—	5.37	—	—	7.13	C.O
20	5.37	10.75	-30.75	52.13	Final Values.

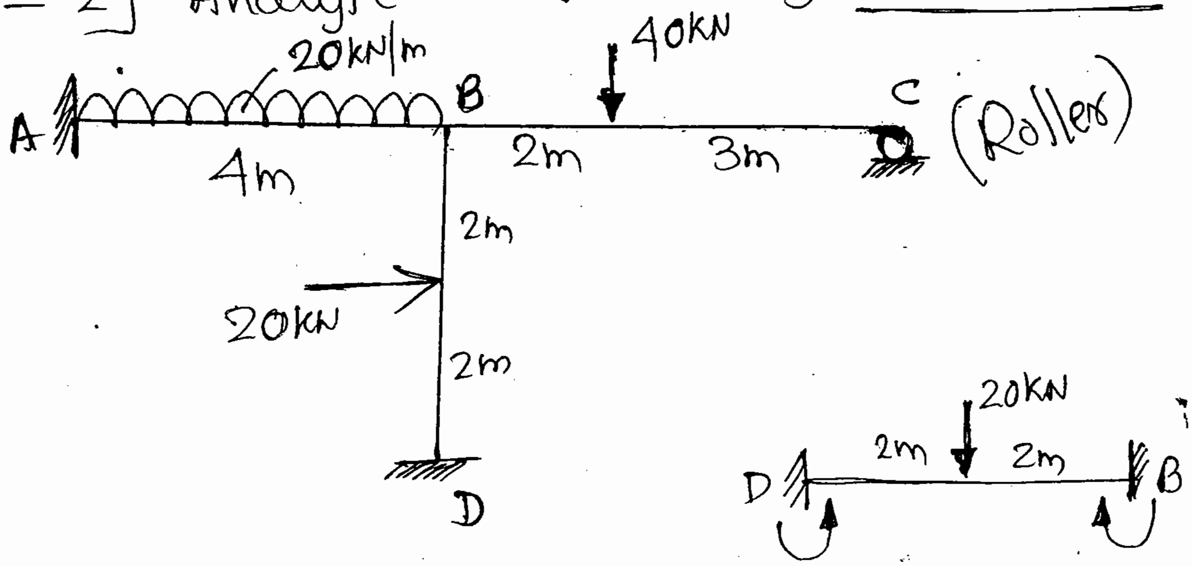
At "B" $\boxed{M_{BA} + M_{BC} + M_{BB'} = 0}$

$$0 - 45 + 20 = -25$$

Refer S.D. method for Reaction, SFD & BMD.

=x=

Eg:- 2] Analyse the frame by M.D. method



(a) FEM :

$M_{FAB} = -\frac{wl^2}{12} = -26.67$, $M_{FBA} = +26.67$

$M_{FBC} = -\frac{Wab^2}{l^2} = -28.8$, $M_{FCB} = +\frac{Wa^2b}{l^2} = +19.2$

$M_{FDB} = -\frac{Wl}{8} = -10$; $M_{FBD} = +10$

(b) D.F : (For Intermediate)

		K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$I/l = I/4 = 0.25I$		0.38
	BC	$\frac{3}{4}(I/l) = \frac{3}{4}(I/5) = 0.15I$	0.65I	0.24
	BD	$I/l = I/4 = 0.25I$		0.38

(c)

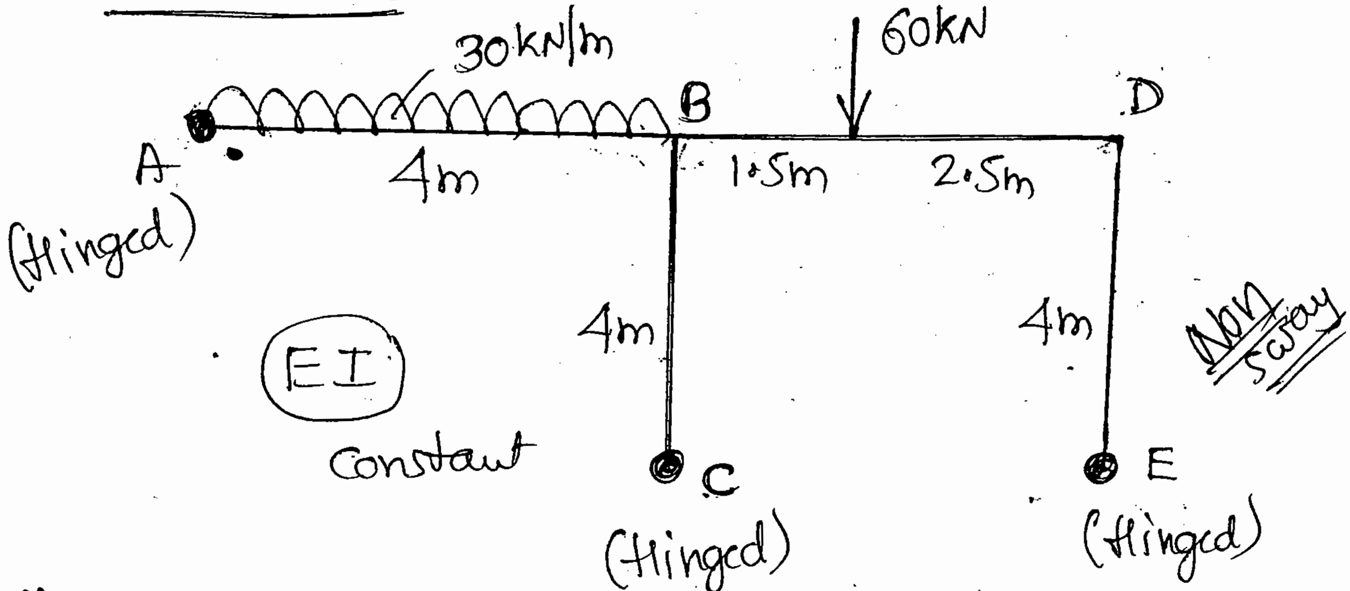
"C" Roller

AB	BA	BD	DB	BC	CB	Member
	0.38	0.38		0.24		DF
−26.67	26.67	+10	−10	−28.8	19.2	FEM
				−9.60	−19.2	Release Ⓒ C.O.
−26.67	26.67	10	−10	−38.40	0	Initial Values.
0.33	0.66	0.66	0.33	0.41	0	Bal C.O
−26.33	27.33	10.66	−9.67	−37.99	0	Final Values.
					0	

Refer S.D. Notes For BMD.

At "B" $M_{BA} + M_{BC} + M_{BD} = 0$

Eg:- 3] Analyse the frame shown by MD method.



Sol^n (a) FEM

$M_{FAB} = -\frac{\omega l^2}{12} = -40$, $M_{FBA} = +40$

$M_{FBD} = -35.16$, $M_{FDB} = +21.10$ kN-m

(b) D.F (For Intermediate)

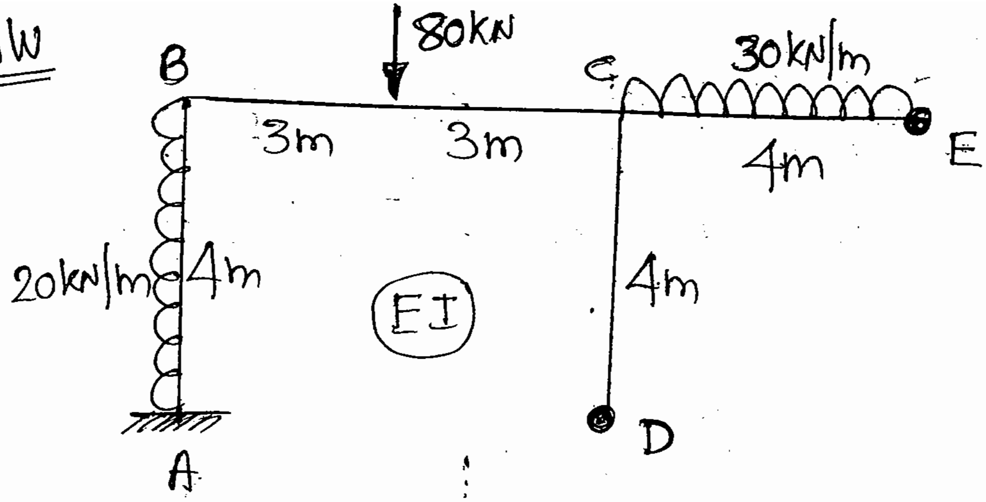
		K	Σk	$DF = \frac{K}{\Sigma k}$
B	BA	$\frac{3}{4}(I/4) = 0.187I$	0.625I	0.30
	BC	$\frac{3}{4}(I/4) = 0.187I$		0.30
	BD	$I/4 = 0.25I$		0.40
D	DB	$I/4 = 0.25I$	0.437I	0.57
	DE	$\frac{3}{4}\left(\frac{I}{4}\right) = 0.187I$		0.43.

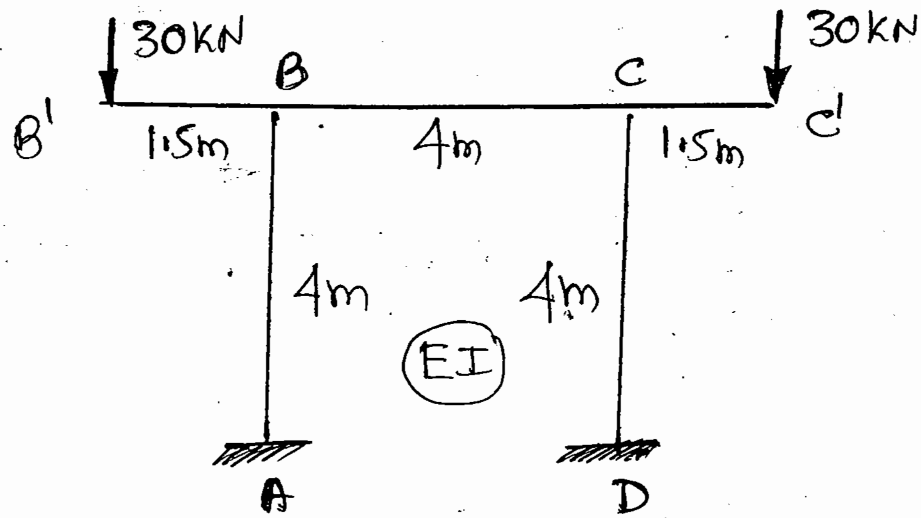
M.D. Table :

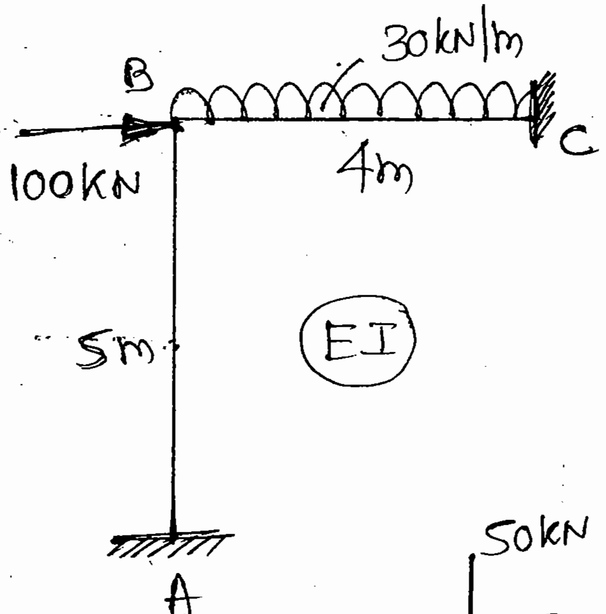
AB	BA	BC	CB	BD	DB	DE	ED	Member
	0.30	0.30		0.40	0.57	0.43		DF
-40	40	0	0	-35.16	21.10	0	0	FEM
+40	20							Release @ (A) C.O
0	60	0	0	-35.16	21.10	0	0	Initial
	-7.45	-7.45		-9.94	-12.03	-9.07		Bal
0			0	-6.01	-4.97		0	C.O
	1.80	1.80		2.40	2.83	2.14		Bal
0			0	1.42	1.20		0	C.O
	-0.43	-0.43		-0.57	-0.68	-0.52		Bal.
0			0	-0.34	-0.28		0	C.O
	0.10	0.10		0.14	0.16	0.12		Bal
0			0	0.08	0.07		0	C.O
	-0.02	-0.02		-0.04	-0.04	-0.03		Bal
0	54	-6	0	-48.02	7.36	-7.36	0	Final

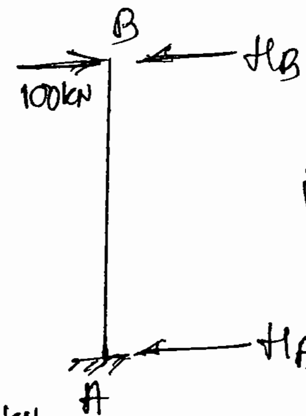
At "B" → $M_{BA} + M_{BC} + M_{BD} = 0$ | At "D" $M_{DB} + M_{DE} = 0$

HW

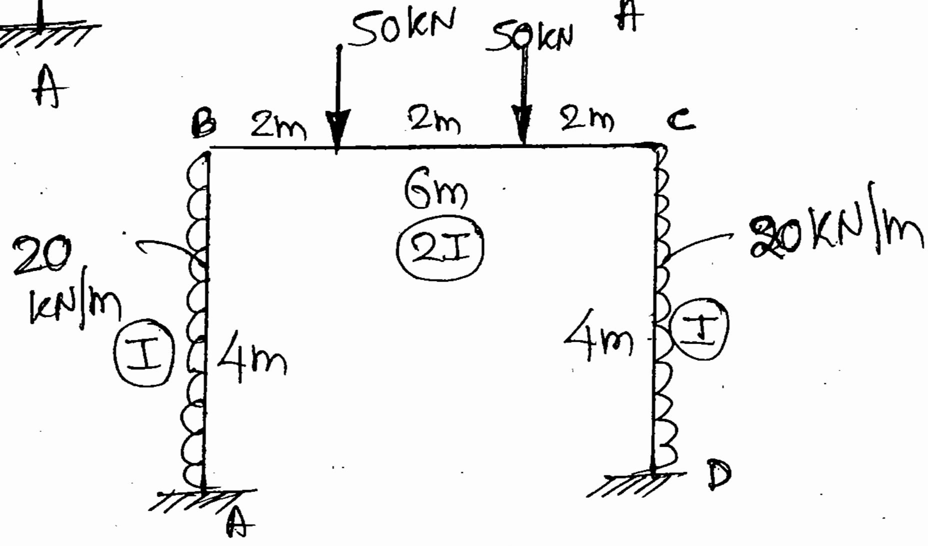








100 kN load is used only at the time of SFD.



MODULE - 3

KANIS METHODS

Procedure:
1) FEM
2) Rotation factor (U)
3) Kani's Box & Rotation moment (m')
4) Final moments
5) Diagrams

1) FEM:

Refer Unit-2 → Slope Deflection method.

2) Rotation factor (U):

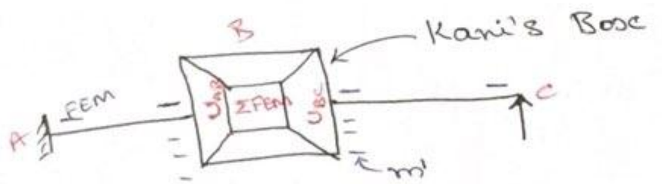
(For intermediate Support joints)

Joint	Member	Relative Stiffness (K)	ΣK	$U = -\frac{1}{2} \cdot \frac{K}{\Sigma K}$

→ Relative Stiffness (K)

(Refer M.D. method notes - Unit-3)

3) Kani's Box & Rotation moment (m'):



(*) $m' = U\left[\Sigma FEM + \Sigma \text{ Far end Rotation moment}\right]$

⇒ @ Simple, Roller (or) Hinge Support

→ Add equal & opposite FEM

→ Carry ~~50%~~ half of the moment from the added value.

→ Then Calculate ΣFEM

⇒ For Overhang Portion, no need of Rotation factor & Kani's Box.

4) Final Moments:

$M = (\Sigma FEM + 2 \times$ Near end rotation moment $+ 1 \times$ Far end rotation moment$)$

5) Diagrams:

Refer Unit-2, Slope Deflection notes.

(*) Types of Problems:

I → Continuous Beam.

II → Continuous Beam with deflection.

III → Non-Sway frames.

(*) Important Formulas:

→ Rotation factor, $U = -\frac{1}{2} \times \frac{K}{\Sigma K}$

→ Rotation moment, $m' = U[\Sigma FEM + \Sigma$ Far end Rotation moment$]$

→ Final moment, $M = FEM + 2 \times$ Near end Rotation moment $+ 1 \times$ Far end Rotation moment.

Kani's Method

(i) Rotation Factor = $\boxed{U = \left(-\frac{1}{2}\right)\frac{K}{\Sigma K}}$

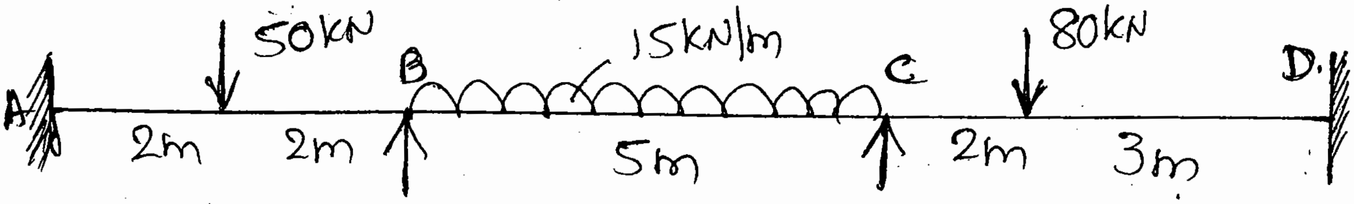
(ii) Rotation Moment :

$$\boxed{m'_{AB} = U\left[\Sigma M_F + \Sigma \text{ Far end Rotation Moment}\right]}$$

(iii) Final Moment

$$\boxed{M = F.E.M + 2\left(\text{Near End Rotation Moment}\right) + \left(\text{Far end Rotation Moment}\right)}$$

Eg:- 1] Analyse the beam shown by Kani's method, Draw BMD.



2012 (a) FEM

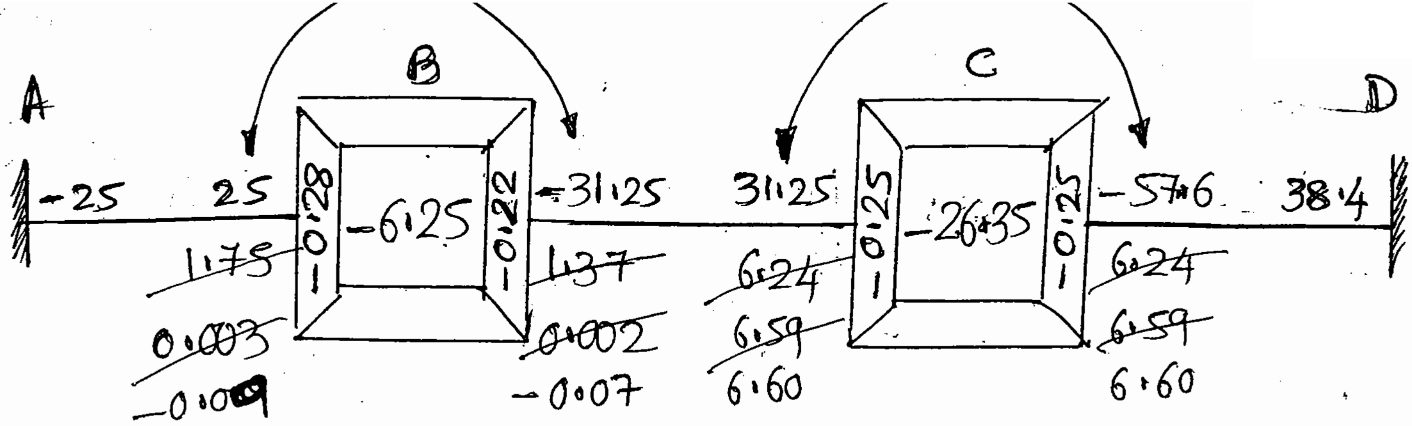
$M_{FAB} = -25$ kN-m, $M_{FBA} = +25$

$M_{FBC} = -31.25$, $M_{FCB} = +31.25$

$M_{FCD} = -57.6$, $M_{FDC} = 38.4$.

(b) Rotation Factor (For Intermediate)

		K	ΣK	$U = \left(-\frac{1}{2}\right)\frac{K}{\Sigma K}$
B	BA	$I/4 = 0.25I$	$0.45I$	-0.28
	BC	$I/5 = 0.20I$		-0.22
C	CB	$I/5 = 0.20I$	$0.4I$	-0.25
	CD	$I/5 = 0.20I$		-0.25



Rotation Moment $\boxed{m'_{AB} = U\left[\sum M_F + \sum \text{Far end Rotation moment}\right]}$

Trial (1)

$$m'_{BA} = -0.28\left(-6.25 + 0\right) = 1.75$$

$$m'_{BC} = -0.22\left(-6.25 + 0\right) = 1.37$$

$$m'_{CB} = -0.25\left(-26.35 + 1.37\right) = 6.24$$

$$m'_{CD} = -0.25\left(-26.35 + 1.37\right) = 6.24$$

Trial (2)

$$m'_{BA} = -0.28\left(-6.25 + 6.24\right) = 0.003$$

$$m'_{BC} = -0.22\left(-6.25 + 6.25\right) = 0.002$$

$$m'_{CB} = -0.25\left(-26.35 + 0.002\right) = 6.59$$

$$m'_{CD} = -0.25\left(-26.35 + 0.002\right) = 6.59$$

Trial (3)

$$m'_{BA} = -0.28\left(-6.25 + 6.59\right) = -0.09$$

$$m'_{BC} = -0.22\left(-6.25 + 6.59\right) = -0.07$$

$$m'_{CB} = -0.25\left(-26.35 - 0.07\right) = 6.60$$

$$m'_{CD} = -0.25\left(-26.35 - 0.07\right) = 6.60$$

Final Moment

$$M = FEM + 2\left(\begin{matrix}\text{Near End} \\ \text{Rotation} \\ \text{moment}\end{matrix}\right) + 1\left(\begin{matrix}\text{Far end} \\ \text{Rotation} \\ \text{moment}\end{matrix}\right)$$

$M_{AB} = -25 + 2(0) - 0.09 = -25.09$ kN-m ↻

$M_{BA} = +25 + 2(-0.09) + 0 = 24.82$ kN-m ↺

$M_{BC} = -31.25 + 2(-0.07) + 6.60 = -24.79$ kN-m ↻

$M_{CB} = +31.25 + 2(6.60) - 0.07 = 44.38$ " ↺

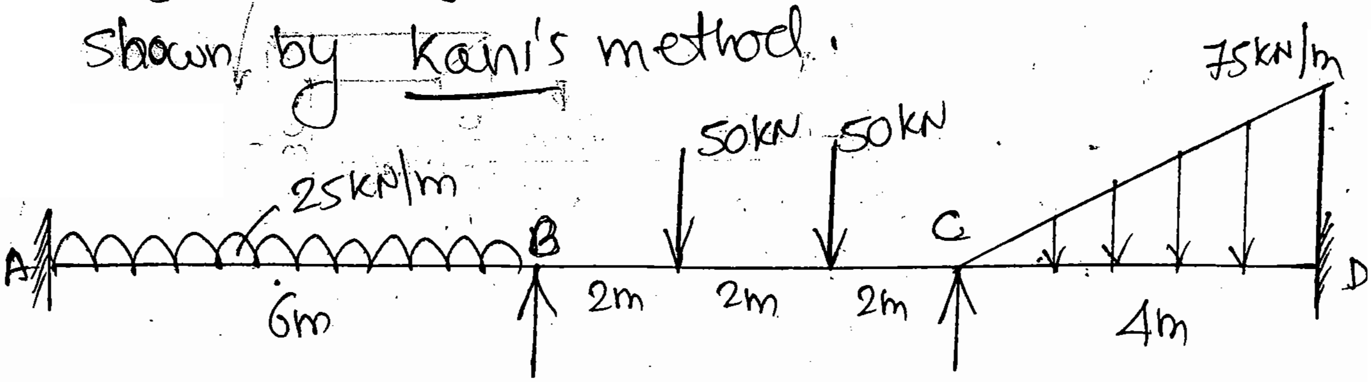
$M_{CD} = -57.6 + 2(6.60) - 0 = -44.40$ " ↻

$M_{DC} = 38.4 + 2(0) + 6.60 = 45$ kN-m ↺

Draw SFD, BMD & EC.

Refer S.D. Notes.

Eg:- Analyse the continuous beam shown by Kani's method.



(a) FEM

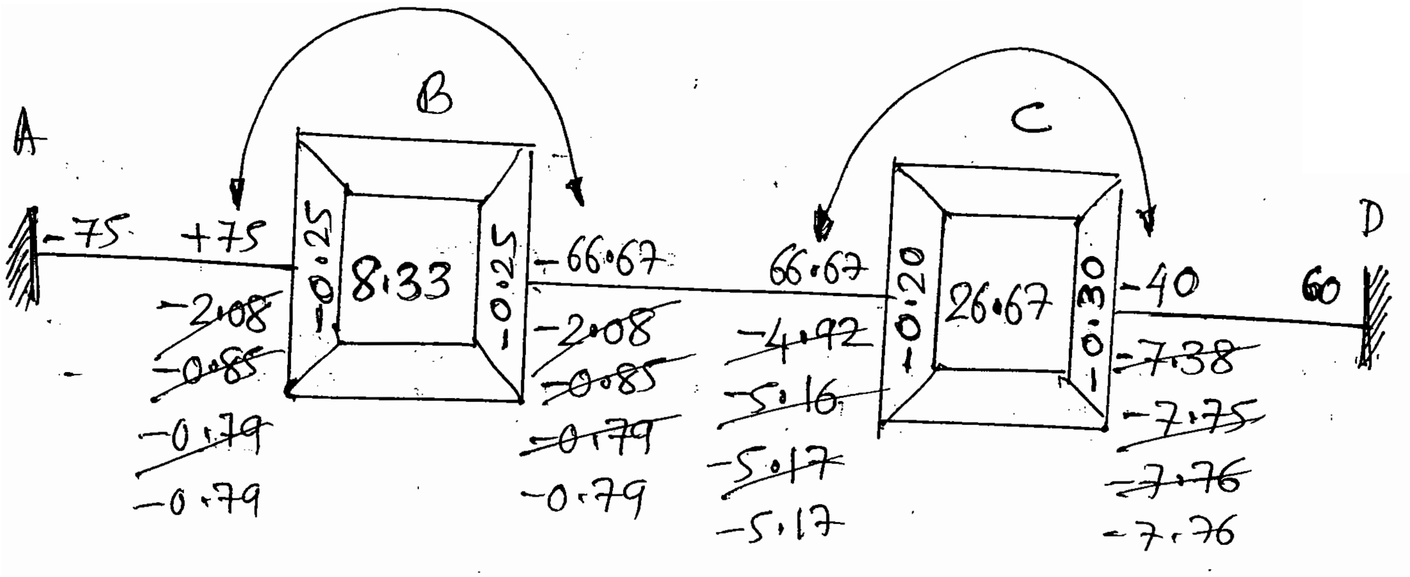
$$M_{FAB} = \frac{-\omega l^2}{12} = -75, \quad M_{FBA} = +75$$

$$M_{FBC} = -\frac{Wab^2}{l^2} = -66.67 \text{ kN-m}, \quad M_{FCB} = +66.67 \text{ kN-m}$$

$$M_{FCD} = \frac{-Wl^2}{30} = -40, \quad M_{FDC} = \frac{+\omega l^2}{20} = +60$$

(b) Rotation Factor (For Intermediate)

		K	ΣK	$U = \left(\frac{-1}{2}\right)\frac{K}{\Sigma K}$
B	BA	$I/l = I/6 = 0.167I$	0.334I	−0.25
	BC	$I/l = I/6 = 0.167I$		−0.25
C	CB	$I/l = I/6 = 0.167I$	0.417I	−0.20
	CD	$I/l = \frac{I}{4} = 0.25I$		−0.30



Rotation Moment

$$\boxed{m = U\left[\Sigma M_F + \Sigma \text{ Far end Rotation Moment}\right]}$$ ✓

Trial ①

$$m_{BA} = -0.25\ (8.33 + 0) = -2.08$$

$$m_{BC} = -0.25\ (8.33 + 0) = -2.08$$

$$m_{CB} = -0.20\ (26.67 - 2.08) = -4.92$$

$$m_{CD} = -0.30\ (26.67 - 2.08) = -7.38$$

Trial ②

$$m_{BA} = -0.25(8.33 - 4.92) = -0.85$$

$$m_{BC} = -0.25(8.33 - 4.92) = -0.85$$

$$m_{CB} = -0.20(26.67 - 0.85) = -5.16$$

$$m_{CD} = -0.30(26.67 - 0.85) = -7.75$$

Trial (3)

$M_{BA} = -0.25\,(8.33 - 5.16) = -0.79$

$M_{BC} = -0.25\,(8.33 - 5.16) = -0.79$

$M_{CB} = -0.20\,(26.67 - 0.79) = -5.17$

$M_{CD} = -0.30\,(26.67 - 0.79) = -7.76$

Trial (4)

$M_{BA} = -0.79$ kN-m

$M_{BC} = -0.79$

Final Moment :-

$$\boxed{M = FEM + 2\begin{bmatrix}\text{Near end}\\ \text{Ro. Moment}\end{bmatrix} + \begin{bmatrix}\text{Far End}\\ \text{Ro. moment}\end{bmatrix}}$$

$M_{AB} = -75 + 2(0) - 0.79 = -75.79$ kN-m

$M_{BA} = +75 + 2(-0.79) + 0 = 73.42$ kN-m

$M_{BC} = -66.67 + 2(-0.79) - 5.17 = -73.42$ kN-m

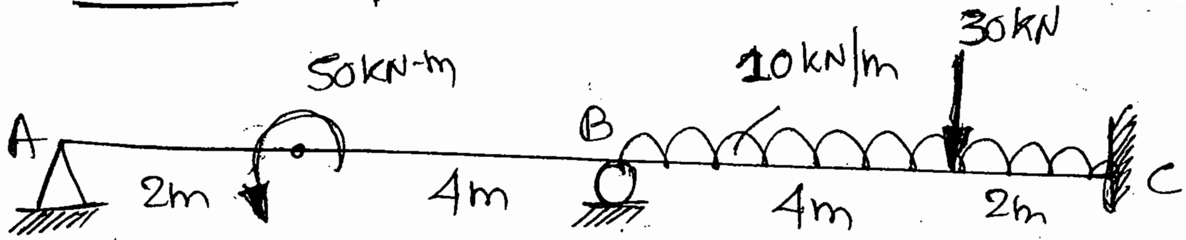
$M_{CB} = +66.67 + 2(-5.17) - 0.79 = 55.54$ kN-m

$M_{CD} = -40 + 2(-7.76) + 0 = -55.52$ kN-m

$M_{DC} = +60 + 2(0) - 7.76 = 52.24$ kN-m

Refer M.D. Notes for SFD & BMD

Eg:-3] Analyse the beam shown by Kani's method



Sol^n (a) FEM

$$M_{FAB} = -\frac{Mb(2a-b)}{l^2} = -\frac{50\times4(2\times2-4)}{6^2} = 0$$

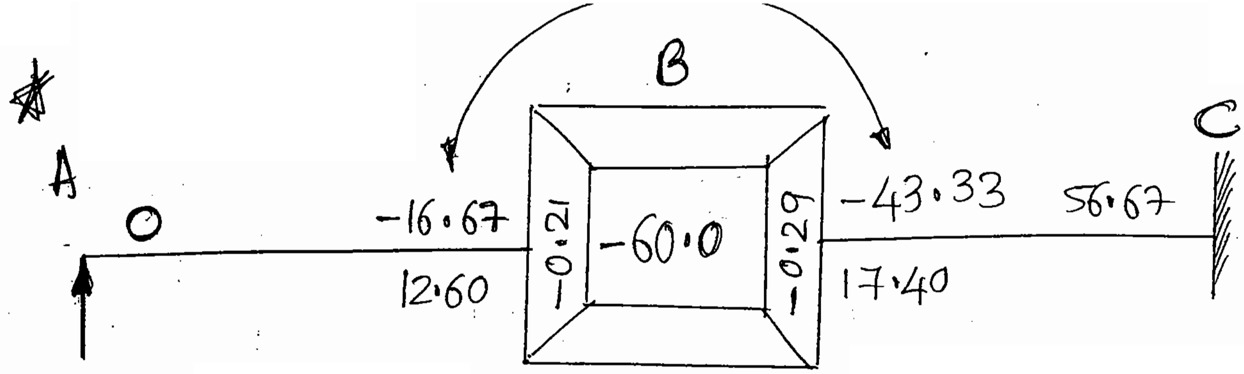
$$M_{FBA} = -\frac{Ma(2b-a)}{l^2} = -\frac{50\times2(2\times4-2)}{6^2} = -16.67$$

$$M_{FBC} = -\frac{wl^2}{12} - \frac{Wab^2}{l^2} = -43.33 \text{ kN-m}$$

$$M_{FCB} = +\frac{wl^2}{12} + \frac{Wa^2b}{l^2} = +56.67$$

(b) Rotation Factor : (For Intermediate)

		k	ΣK	$U = \left(-\frac{1}{2}\right)\frac{K}{\Sigma K}$
B	BA	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{I}{6}\right) = 0.125I$	$0.292I$	-0.21
	BC	$\frac{I}{l} = \frac{I}{6} = 0.167I$		-0.29



Rotation Moment

$$M = U\left[\sum M_F + \sum \text{Rotation far end moment}\right]$$

$$M_{BA} = -0.21\,(-60 + 0) = 12.60$$

$$M_{BC} = -0.29\,(-60 + 0) = 17.40$$

Final Moment :

$$\boxed{M = FEM + 2\begin{pmatrix}\text{Near end}\\ \text{Ro. moment}\end{pmatrix} + \begin{pmatrix}\text{Far end}\\ \text{Ro moment}\end{pmatrix}}$$

$M_{AB} = 0$ ★ If last support is simple or Hinge or Roller the above eq^n is not applicable.

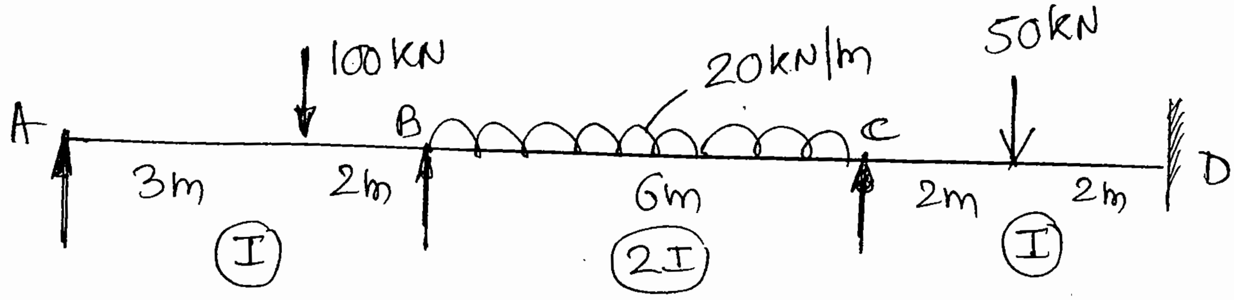
$$M_{BA} = -16.67 + 2(12.60) + 0 = 8.53$$

$$M_{BC} = -43.33 + 2(17.40) + 0 = -8.53 \text{ kN-m}$$

$$M_{CB} = 56.67 + 2(0) + 17.40 = 74.07 \text{ kN-m}$$

Draw BMD & SFD.

Eg:- 4 Analyse the beam shown by kani's method



Sol:

(a) FEM

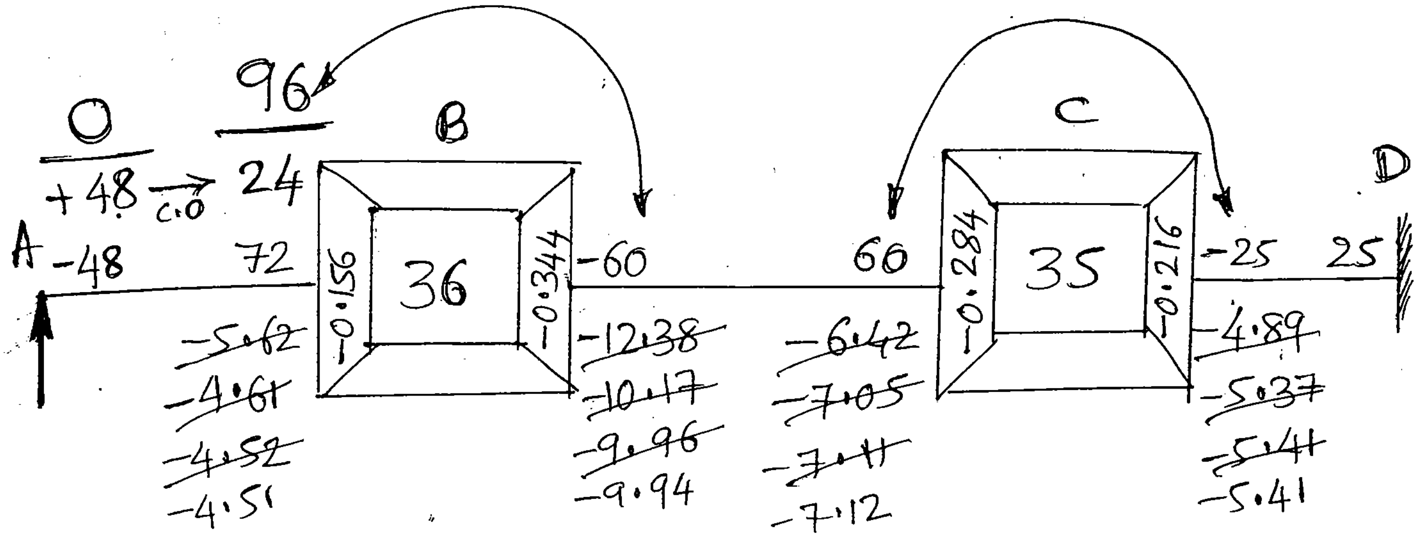
$M_{FAB} = -48$, $M_{FBA} = 72$

$M_{FBC} = -60$, $M_{FCB} = +60$

$M_{FCD} = -25$, $M_{FDC} = 25$.

(b) Rotation Factor :-

		K	ΣK	$U = \left(-\frac{1}{2}\right)\frac{K}{\Sigma K}$
B	BA	$\frac{3}{4}\left(\frac{I}{5}\right) = 0.15I$	0.48I	-0.156
	BC	$\frac{2I}{6} = 0.33I$		-0.344
C	CB	$\frac{2I}{6} = 0.33I$	0.58I	-0.284
	CD	$\frac{I}{4} = 0.25I$		-0.216



Rotation Moment

$$\boxed{M = U\left[\Sigma M_F + \Sigma \text{ Far End Ro. Moment}\right]}$$

Trial-1

$$M_{BA} = -0.156\,(36+0) = -5.62$$

$$M_{BC} = -0.344\,(36+0) = -12.38$$

$$M_{CB} = -0.284\,(35-12.38) = -6.42$$

$$M_{CD} = -0.216\,(35-12.38) = -4.89$$

Final Moment: $\boxed{M = FEM + 2(\text{Near}) + (\text{Far})}$

$M_{AB} = 0$ * The above eq^n is not applicable *

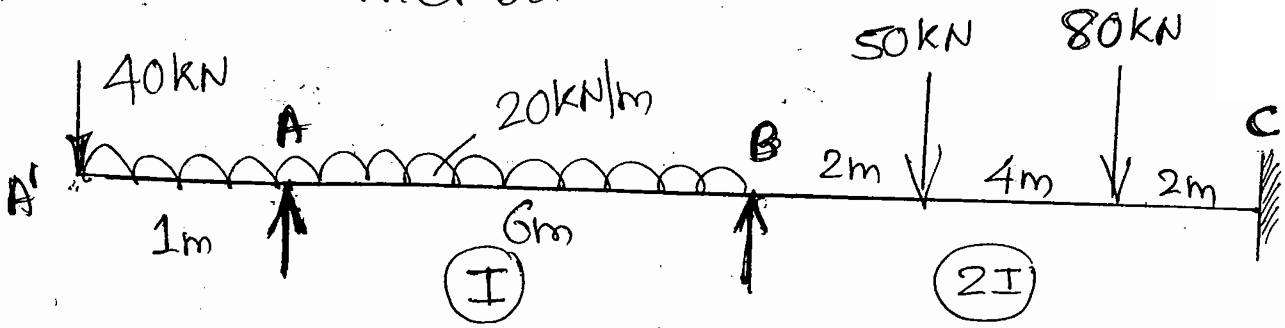
$M_{BA} = 96 + 2(-4.51) + 0 = 86.98$ kN-m

$M_{BC} = -60 + 2(-9.94) - 7.12 = -87.00$ kN-m

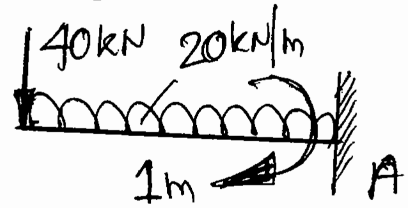
$M_{CB} = +60 + 2(-7.12) - 9.94 = 35.82$ kN-m

$M_{CD} = -35.82$ kN-m, $M_{DC} = 19.59$ kN-m

Eg: 5] Analyse the beam shown by Kani's method.



Sol^n (a) FEM



$M_{AA'} = +40\times1 + 20\times1\times\frac{1}{2} = +50$ kN-m (Final moment)

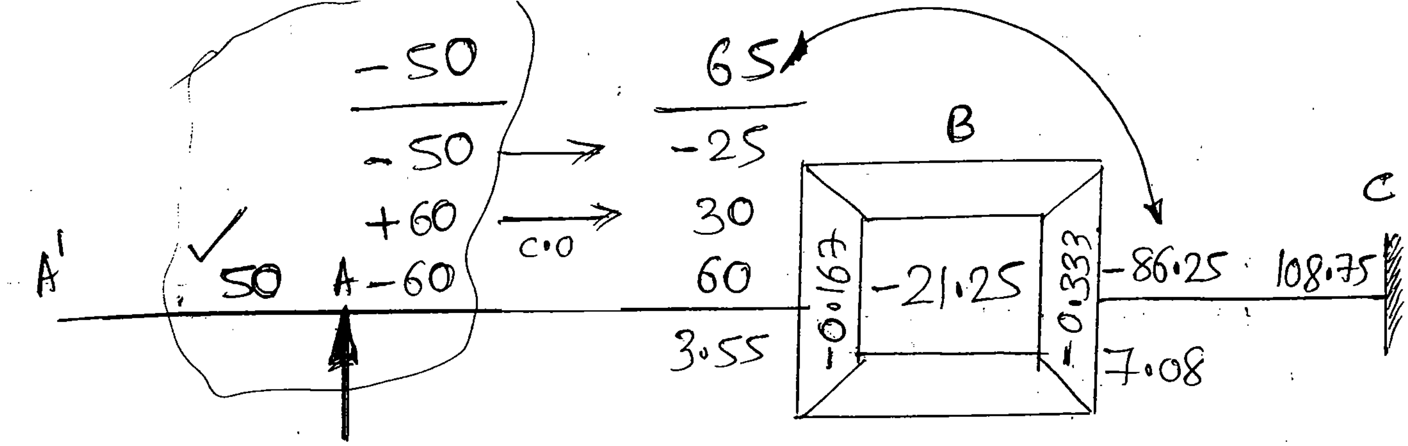
$M_{FAB} = -60$, $M_{FBA} = +60$

$M_{FBC} = -86.25$, $M_{FCB} = 108.75$.

(b) Rotation Factor :

★ The support with overhang portion, ~~ration~~ rotation factor and Kani's Box both are not required. ★

		K	ΣK	$U = \left(\frac{-1}{2}\right)\frac{K}{\Sigma K}$
B	BA	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{I}{6}\right) = 0.125I$	0.375I	-0.167
	BC	$\frac{I}{l} = \frac{2I}{8} = 0.25I$		-0.333



Rotation moment

$$M_{BA} = -0.167(-21.25+0) = 3.55\ \text{kN-m}$$

$$M_{BC} = -0.333(-21.25+0) = 7.08\ \text{kN-m}$$

Final Moment: $\boxed{M = FEM + 2(\text{Near}) + (\text{Far})}$ ✓

$M_{AA'} = 50$ kN-m
$M_{AB} = -50$ kN-m

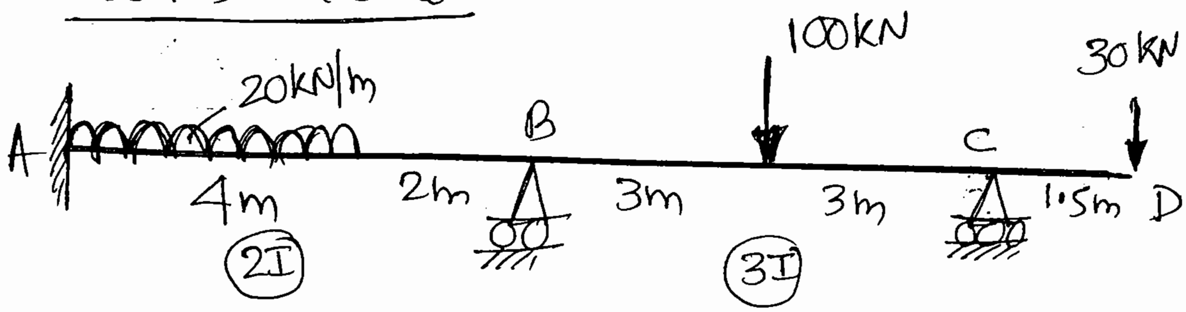
★ For these two the eq^n is not applicable. They are final moments.

$$M_{BA} = 65 + 2(3.55) + 0 = 72.1\ \text{kN-m}$$

$$M_{BC} = -86.25 + 2(7.08) + 0 = -72.10\ \text{kN-m}$$

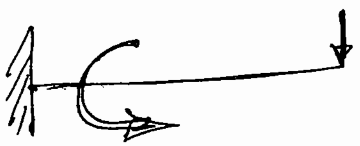
$$M_{CB} = 108.75 + 2(0) + 7.08 = 115.83\ \text{kN-m}$$

Eg:-6] Analyse the beam shown by Kani's method



Sol: (a) $M_{FAB} = -53.33$, $M_{FBA} = +35.56$

$M_{FBC} = -75$, $M_{FCB} = +75$



$M_{CD} = -30 \times 1.5 = -45$ KN-m.

(b) R.F: (Only at "B").

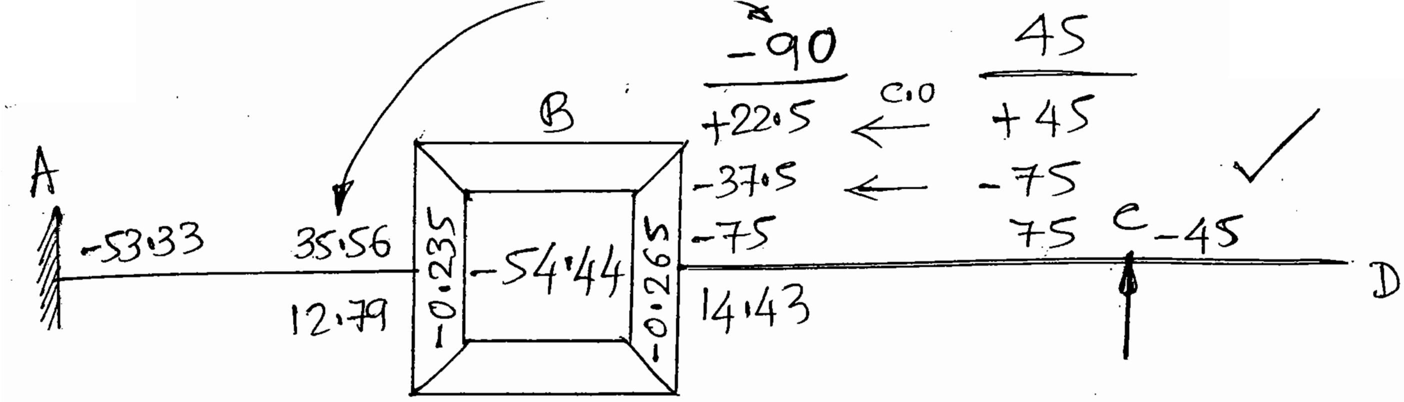
		K	ΣK	U
B	BA	$I/l = 2I/6 = 0.333I$	$0.708I$	-0.235
	BC	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{3I}{6}\right) = 0.375I$		-0.265

☆ Any overhanging moments are final.

$\therefore M_{CD} = -45$ kN-m

$\therefore$ From equillibrium point of view

"M_{CB}" should be $+45$ kN-m



Rotation Moment

$$m = U\left[\Sigma FEM + \Sigma \text{ Farend Ro. Moment}\right]$$

$$m_{BA} = -0.235(-54.44 + 0) = 12.79$$

$$m_{BC} = -0.265(-54.44 + 0) = 14.43$$

Final Moments :

$$M = FEM + 2(\text{Near}) + (\text{Far})$$

$$M_{AB} = -53.33 + 2(0) + 12.79 = -40.54 \text{ kN-m}$$

$$M_{BA} = 35.56 + 2(12.79) + 0 = 61.14 \text{ kN-m}$$

$$M_{BC} = -90 + 2(14.43) + 0 = -61.14 \text{ kN-m}$$

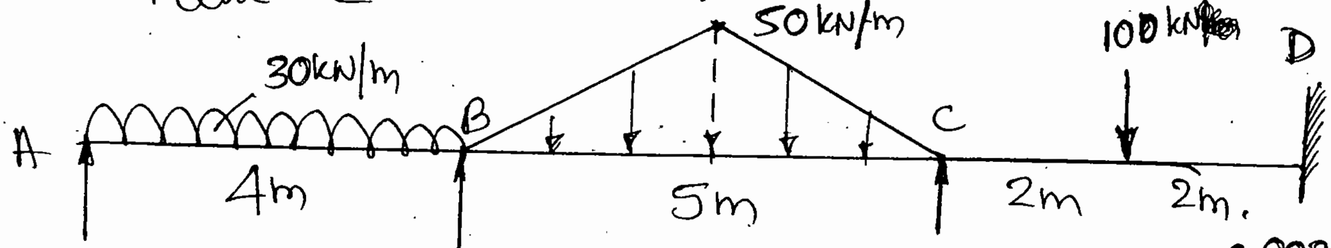
$$M_{CB} = +45 \star$$

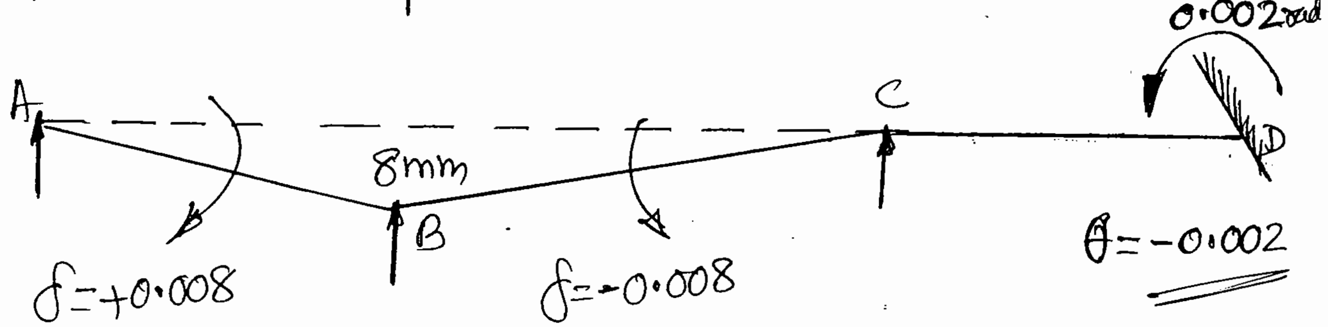
$$M_{CD} = -45 \star$$

Sinking of Support

Eg:- Analyse the beam shown by kani's method. The support "D" rotates by 0.002 rad. anti-clockwise. The support 'B' sinks by 8mm.

Take $E = 210\,GPa$, $I = 0.1\,G\,mm^4$.





$$E = 210 \times 10^9 \times 10^{-6} = 210 \times 10^3\ N/mm^2$$

$$I = 0.1 \times 10^9\ mm^4$$

$$EI = \frac{(210 \times 10^3)(0.1)10^9\ \ N\text{-}mm^2}{(10^3)\ (10^3)^2} = 21000\ kN\text{-}m^2$$

(a) FEM :

$$\text{Additional moment} = -\frac{6EI\delta}{l^2} \quad \text{(sinking)}$$

$$= \frac{4EI\theta}{l} \rightarrow \text{Near end Rotation}$$

$$= \frac{2EI\theta}{l} \rightarrow \text{Far end rotation}$$

$$M_{FAB} = -\frac{wl^2}{12} - \frac{6EI\delta}{l^2} = \frac{-30 \times 4^2}{12} - \frac{6(21000)(0.008)}{4^2}$$

$$= -103.0 \text{ kN-m}$$

$$M_{FBA} = +\frac{wl^2}{12} - \frac{6EI\delta}{l^2} = -23 \text{ kN-m}.$$

$$M_{FBC} = -\frac{5wl^2}{96} - \frac{6EI\delta}{l^2} = \frac{-5(50)5^2}{96} - \frac{6(21000)(-0.008)}{5^2}$$

$$= -24.78$$

$$M_{FCB} = +\frac{5wl^2}{96} - \frac{6EI\delta}{l^2} = 105.42 \text{ kN-m}.$$

Far end

$$M_{FCD} = -\frac{Wl}{8} + \frac{2EI\theta}{l} = \frac{-100 \times 4}{8} + \frac{2(21000)(-0.002)}{4}$$

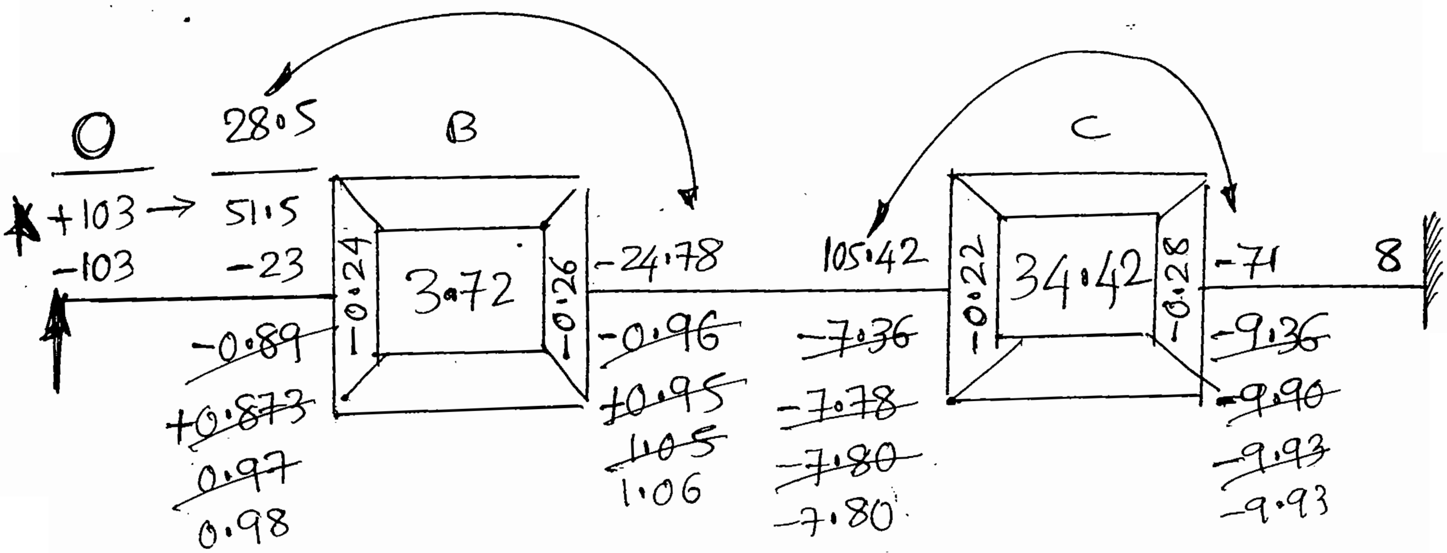
$$= -71 \text{ kN-m}$$

Near

$$M^F_{DC} = +\frac{Wl}{8} + \frac{4EI\theta}{l} = \frac{100 \times 4}{8} + \frac{4(21000)(-0.002)}{4}$$

$$= +8$$

		K	ΣK	U
B	BA	$\frac{3}{4}(I/4) = 0.1875I$	$0.3875I$	-0.24
	BC	$I/5 = 0.2I$		-0.26
C	CB	$I/5 = 0.2I$	$0.45I$	-0.22
	CD	$I/4 = 0.25I$		-0.28



$$m = U\left[\Sigma \text{FEM} + \Sigma \text{Far end Ro. moment}\right]$$

Trial (1)

$m_{BA} = -0.24\,(3.72 + 0) = -0.89$

$m_{BC} = -0.26\,(3.72 + 0) = -0.96$

$m_{CB} = -0.22\,(34.42 - 0.96) = -7.36$

$m_{CD} = -0.28\,(34.42 - 0.96) = -9.36$

Final Moment

$M_{AB} = 0$ ★

$M_{BA} = 28.5 + 2(0.98) + 0 = 30.46$ kN-m

$M_{BC} = \qquad = -30.46$ kN-m

$M_{CB} = \qquad = 90.88$ kN-m

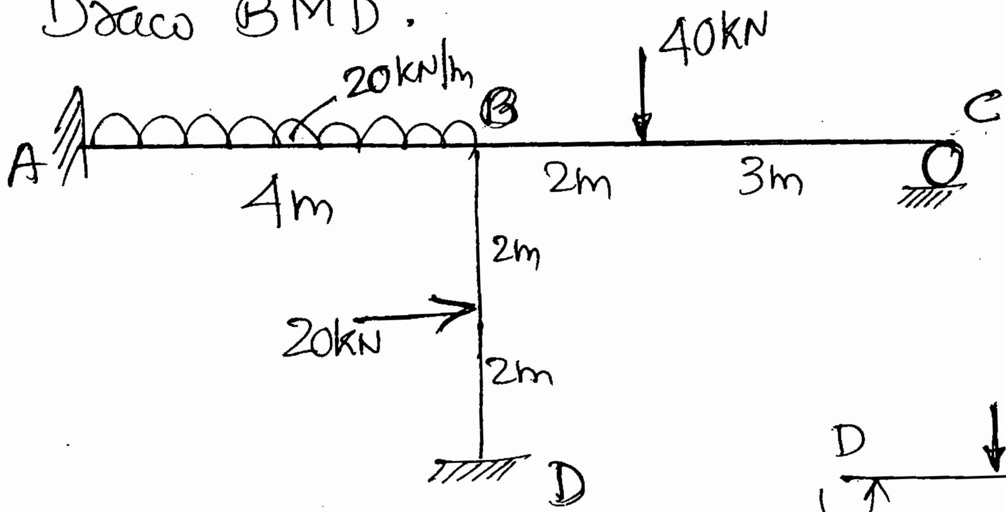
$M_{CD} = \qquad = -90.86$ kN-m

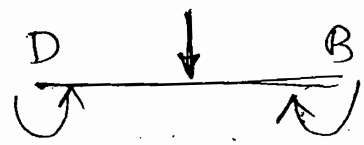
$M_{DC} = \qquad = -1.93$ kN-m

Non Sway Frames

Eg:- Analyse the frame by kani's method.

Draw BMD.





Sol²

(a) FEM

$M_{FAB} = -26.67$, $M_{FBA} = +26.67$

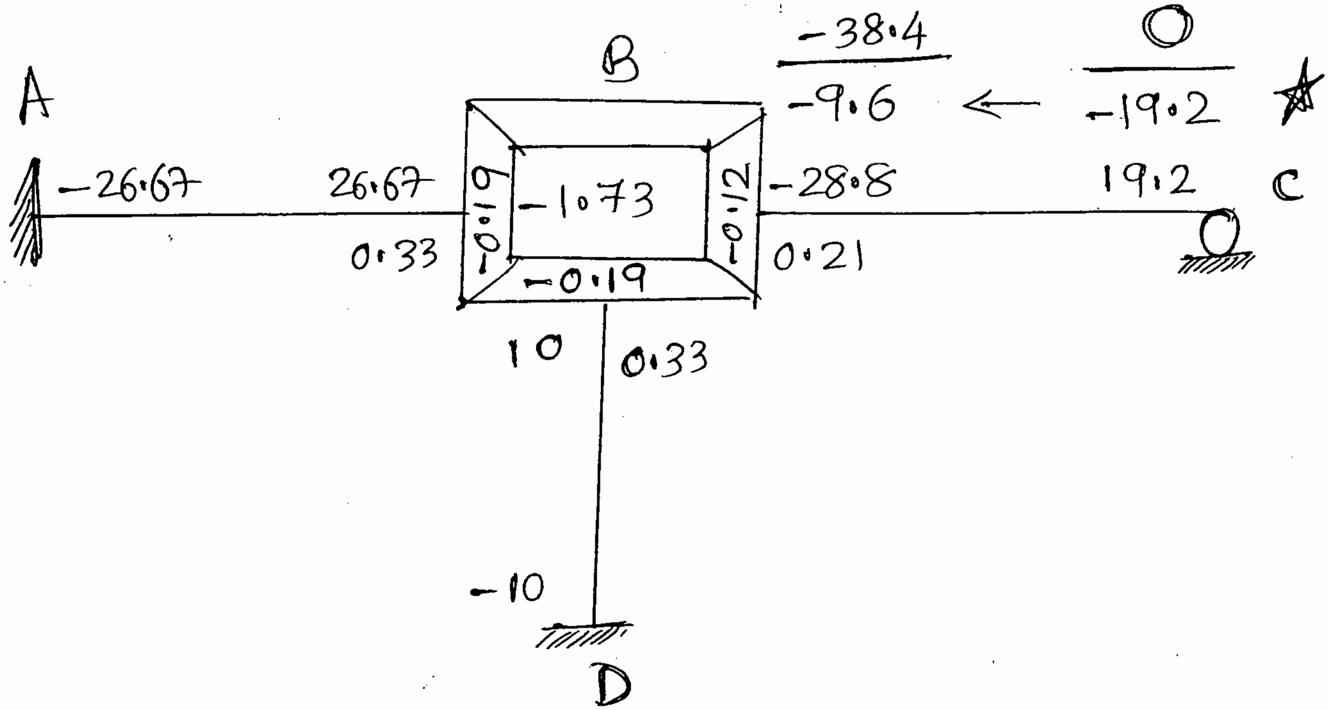
$M_{FBC} = -28.8$, $M_{FCB} = +19.20$

$M_{FDB} = -10$, $M_{FBD} = +10$

(b) R.F

		K	ΣK	U
	BA	$I/4 = 0.25I$		-0.19
B	BC	$\frac{3}{4}\left(I/5\right) = 0.15I$	$0.65I$	-0.12
	BD	$I/4 = 0.25I$		-0.19

At B → 26.67 − 38.4 + 10 = (−1.73)
BA BC BD



$$m_{BA} = -0.19(-1.73+0) = 0.33$$

$$m_{BC} = -0.12(-1.73+0) = 0.21$$

$$m_{BD} = -0.19(-1.73+0) = 0.33$$

Final :-

$$M_{AB} = -26.67 + 2(0) + 0.33 = -26.34 \text{ KN-m}$$

$$M_{BA} = 26.67 + 2(0.33) + 0 = 27.33 \text{ "}$$

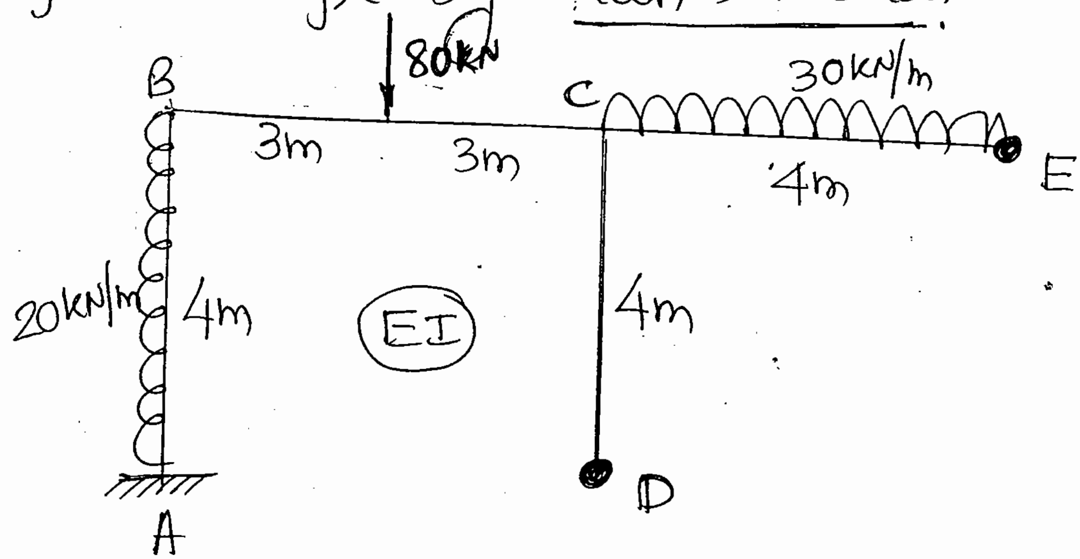
$$M_{BC} = -38.4 + 2(0.21) + 0 = -37.98 \text{ "}$$

$$M_{CB} = 0 \star$$

$$M_{BD} = 10 + 2(0.33) + 0 = 10.66 \text{ "}$$

$$M_{DB} = -10 + 2(0) + 0.33 = -9.67 \text{ "}$$

Eg:- Analyse by Kani's Method



Sol^n

(a) FEM

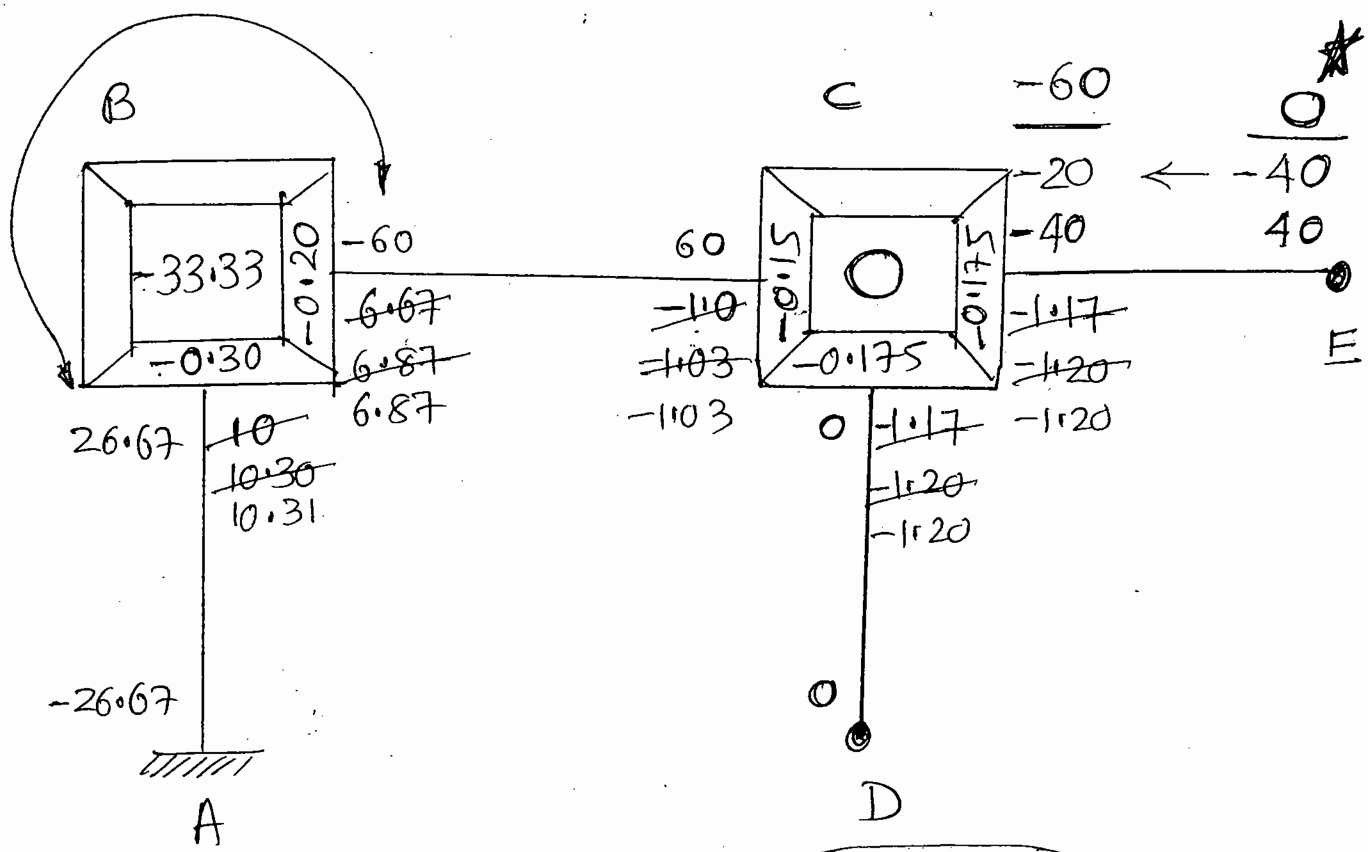
$M_{FAB} = -26.67$, $M_{FBA} = +26.66$ kN

$M_{FBC} = -60$, $M_{FCB} = +60$

$M_{FCE} = -40$, $M_{FEC} = +40$

(b) Rotation Factor (For "B" & "C")

		K	ΣK	$U = \left(-\frac{1}{2}\right)\frac{K}{\Sigma K}$
B	BA	$I/4 = 0.25I$	$0.416I$	-0.3
	BC	$I/6 = 0.167I$		-0.2
C	CB	$I/6 = 0.167I$	$0.542I$	-0.15
	CD	$\frac{3}{4}\left(\frac{I}{4}\right) = 0.1875I$		-0.175
	CE	$\frac{3}{4}\left(\frac{I}{4}\right) = 0.1875I$		-0.175



$$\boxed{m = U\left[\sum FEM + \sum \text{Far end Rot moment}\right]}$$

Final Moment

$M_{AB} = -26.67 + 2(0) + 10.31 = -16.36$

$M_{BA} = 26.67 + 2(10.31) + 0 = 47.29$

$M_{BC} = -60 + 2(6.87) - 1.03 = -47.29$

$M_{CB} = 60 + 2(-1.03) + 6.87 = 64.81$

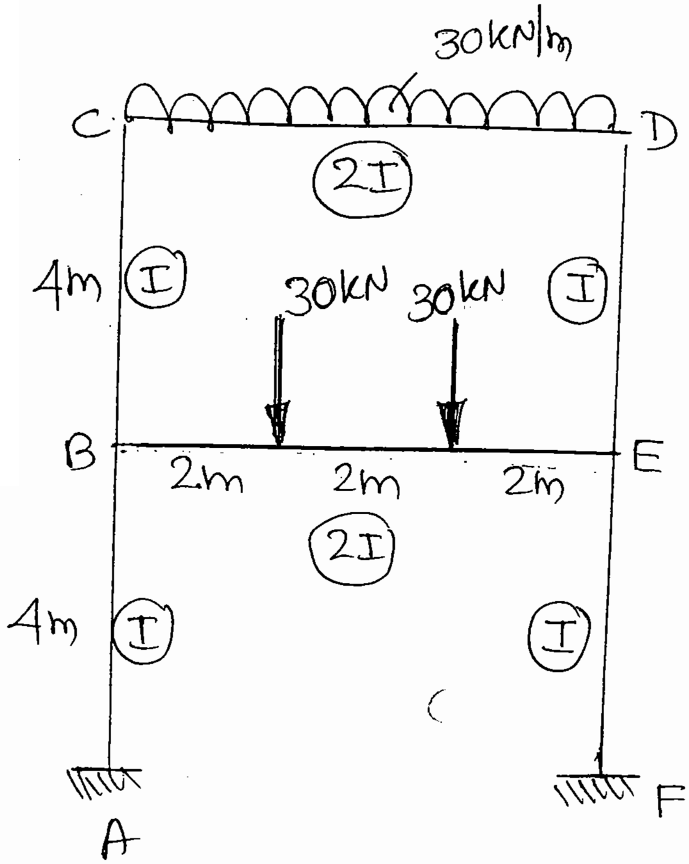
$M_{CD} = 0 + 2(-1.20) + 0 = -2.40$

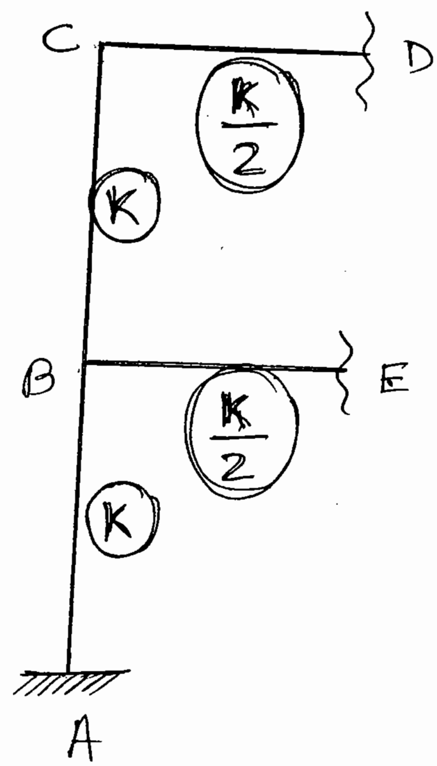
$M_{DC} = 0$

$M_{CE} = -60 + 2(-1.20) + 0 = -62.40$ kN-m

Draw BMD.

Eg:- Analyse the frame shown by Kani's method





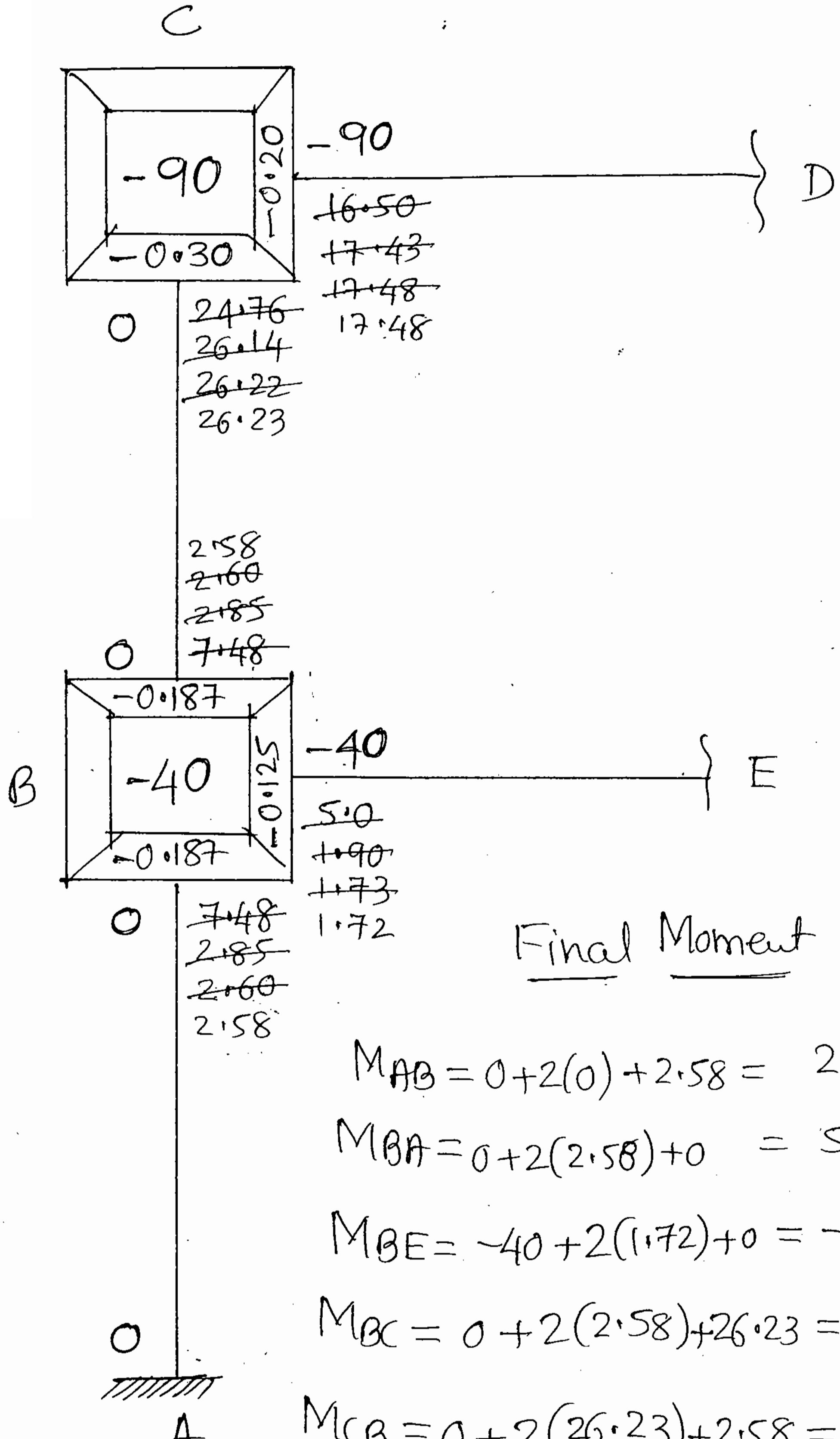
(a) FEM

$$M_{FBE} = \frac{-Wab^2}{l^2} = -\left[\frac{30\times2\times4^2}{6^2} + \frac{30\times4\times2^2}{6^2}\right] = -40$$

$$M_{FCD} = \frac{-wl^2}{12} = -90$$

(b) R.F (Only For "B" & "C")

		K	ΣK	υ
B	BA	$K = I/l = I/4 = 0.25I$	0.667I	−0.187
	BC	$K = I/l = I/4 = 0.25I$		−0.187
	BE	$\frac{K}{2} = \frac{1}{2}\left(\frac{I}{l}\right) = \frac{1}{2}\left(\frac{2I}{6}\right) = 0.167I$		−0.125
C	CB	$K = I/l = I/4 = 0.25I$	0.417I	−0.30
	CD	$\frac{K}{2} = \frac{1}{2}\left(\frac{I}{l}\right) = \frac{1}{2}\left(\frac{2I}{6}\right) = 0.167I$		−0.20



Final Moment

$M_{AB} = 0 + 2(0) + 2.58 = 2.58$ kN-m

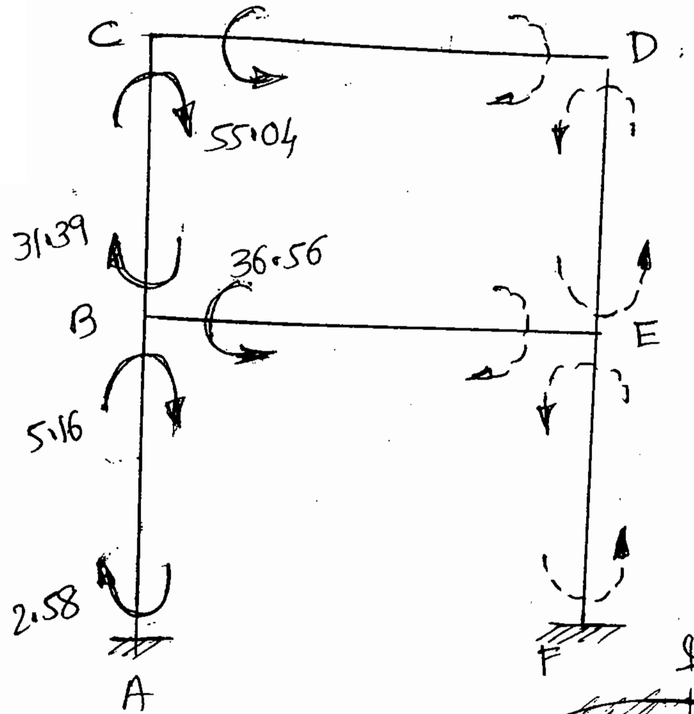
$M_{BA} = 0 + 2(2.58) + 0 = 5.16$ kN-m

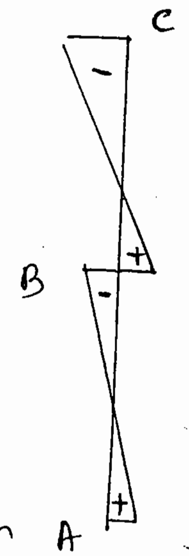
$M_{BE} = -40 + 2(1.72) + 0 = -36.56$ "

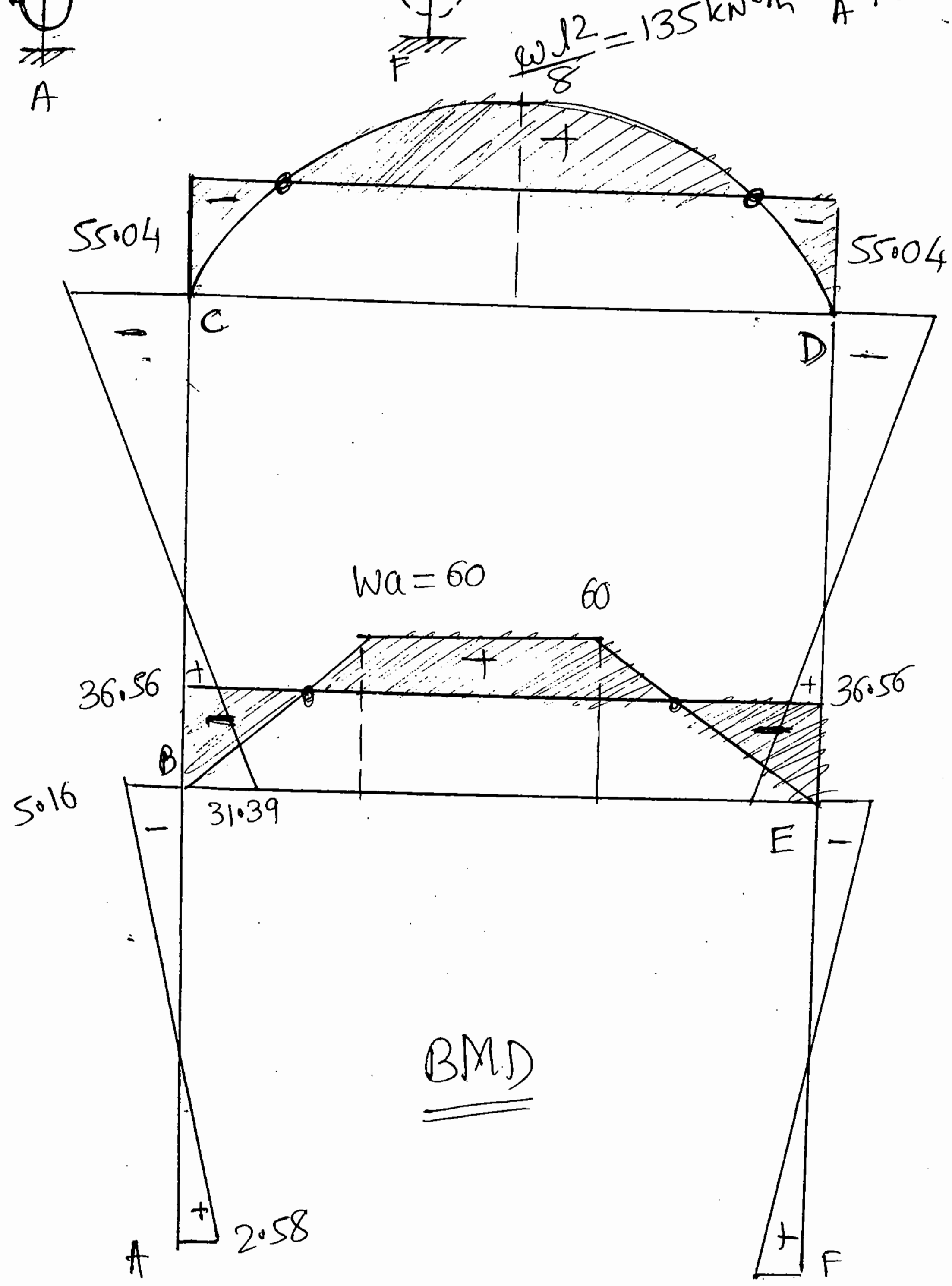
$M_{BC} = 0 + 2(2.58) + 26.23 = 31.39$ "

$M_{CB} = 0 + 2(26.23) + 2.58 = 55.04$ "

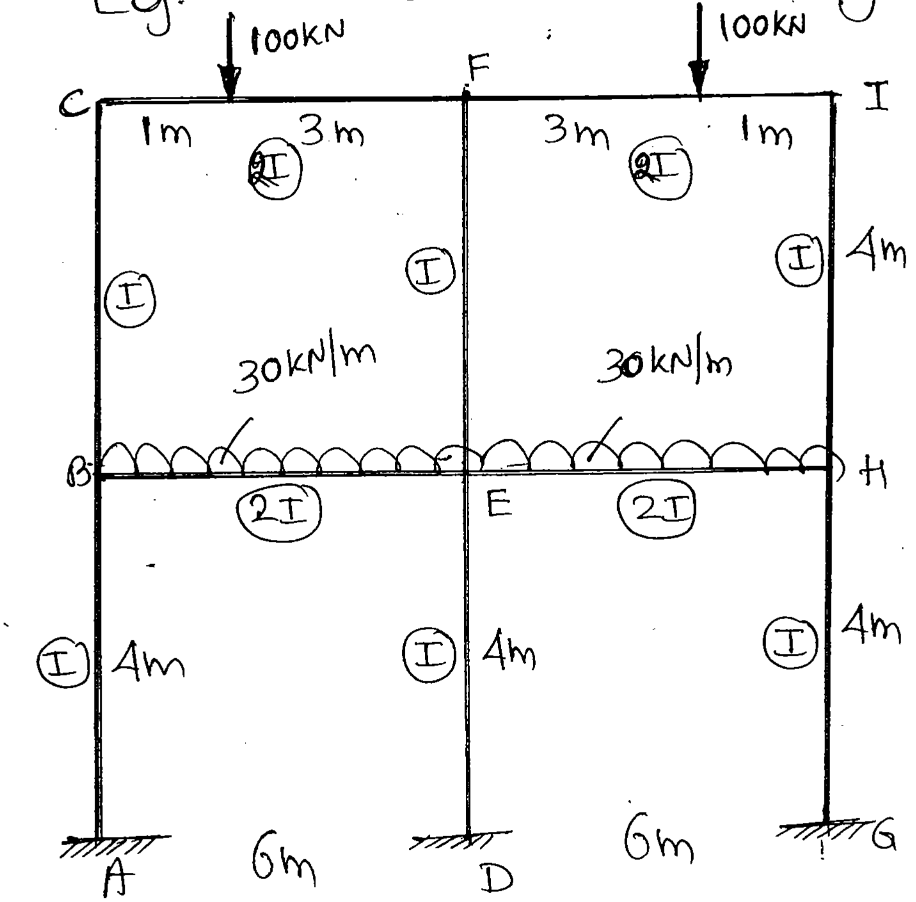
$M_{CD} = -90 + 2(17.48) + 0 = -55.04$ "

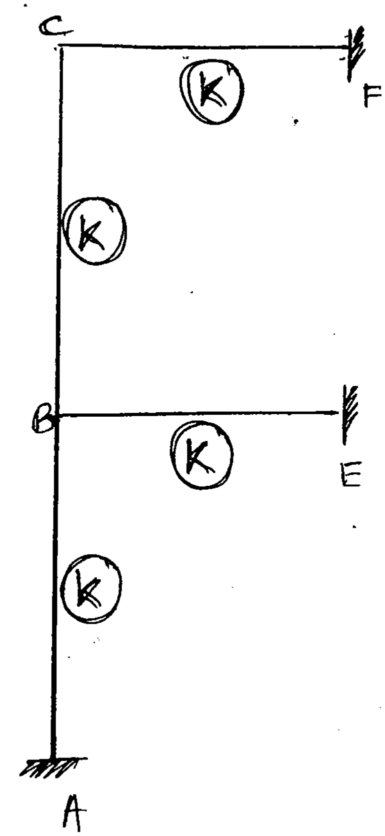

C
D
55.04
31.39
36.56
B
E
5.16
2.58
A
F


C
B
A
+
-


wl²/8 = 135 kN-m
55.04
55.04
C
D
Wa = 60
60
36.56
36.56
B
5.16
31.39
E
2.58
A
F
BMD

Eg:- Analyse the frame by Kani's



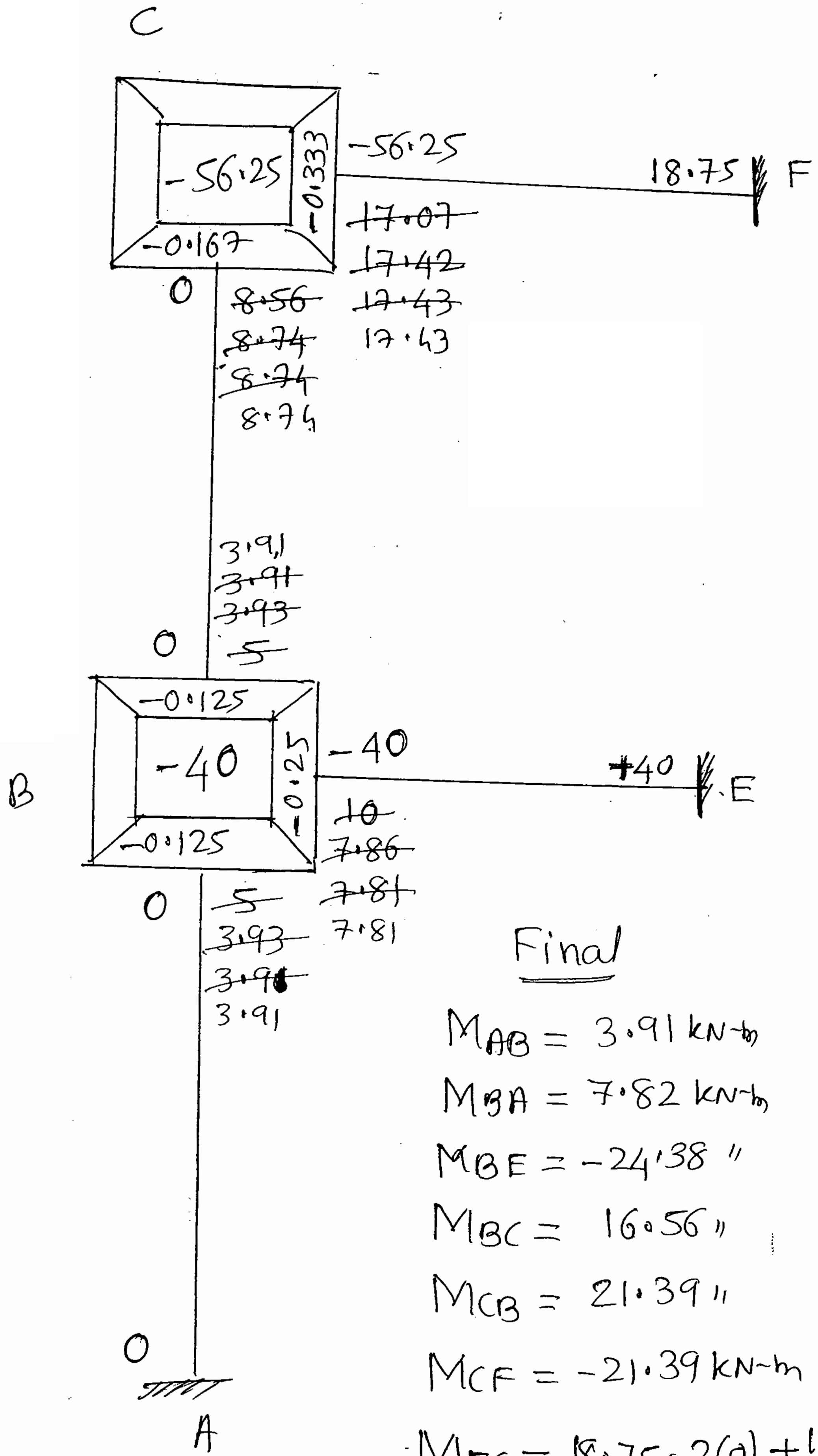


Sol^n (a) FEM

$M_{FBE} = -40$ kN-m, $M_{FCF} = \frac{-Wab^2}{l^2} = -56.25$ kN-m

(b) R.F (Only at B & C) $M_{FFC} = \frac{Wa^2b}{l^2} = 18.75$

		K	ΣK	U
B	BA	K = I/4 = 0.25 I	1.0 I	-0.125
	BE	K = 2I/4 = 0.5 I		-0.25
	BC	K = I/4 = 0.25 I		-0.125
C	CB	K = I/4 = 0.25 I	0.75 I	-0.167
	CF	K = 2I/4 = 0.5 I		-0.333



Final

$M_{AB} = 3.91 \text{ kN-m}$

$M_{BA} = 7.82 \text{ kN-m}$

$M_{BE} = -24.38$ "

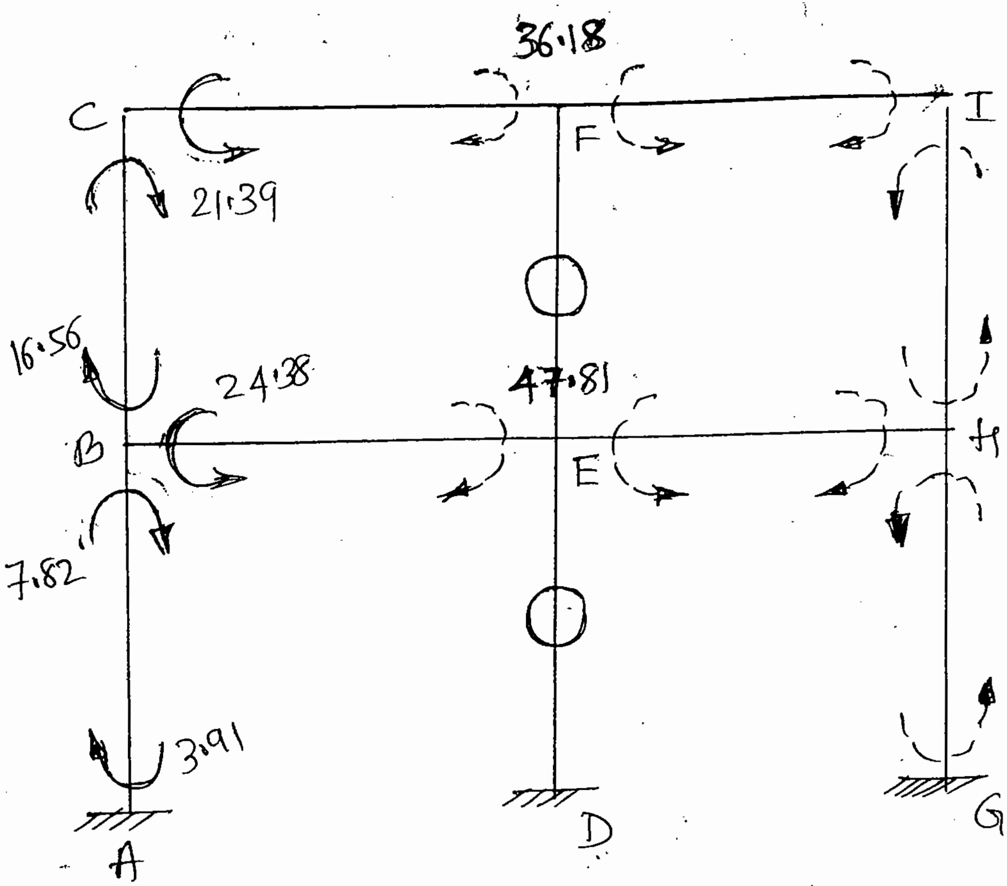
$M_{BC} = 16.56$ "

$M_{CB} = 21.39$ "

$M_{CF} = -21.39 \text{ kN-m}$

$M_{FC} = 18.75 + 2(0) + 17.43 = 36.18$

$M_{EB} = 40 + 2(0) + 7.81 = 47.81$


36.18
C
F
I
21.39
16.56
24.38
47.81
B
E
H
7.82
3.91
A
D
G

MODULE-4

FLEXIBILITY MATRIX METHOD

The systematic development of consistent deformation method in the matrix form has lead to flexibility matrix method. The method is also called force method. Since the basic unknowns are the redundant forces in the structure.

This method is exactly opposite to stiffness matrix method.

The flexibility matrix equation is given by

$$[P]\ [F] = \{[\Delta] - [\Delta_L]\}$$

$$[P] = [F]^{-1}\{[\Delta] - [\Delta_L]\}$$

Where,

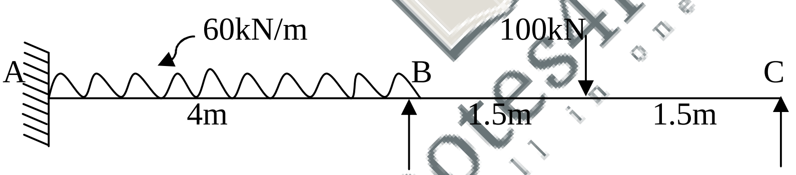
$[P]$ = Redundant in matrix form

$[F]$ = Flexibility matrix

$[\Delta]$ = Displacement at supports

$[\Delta_L]$= Displacement due to load

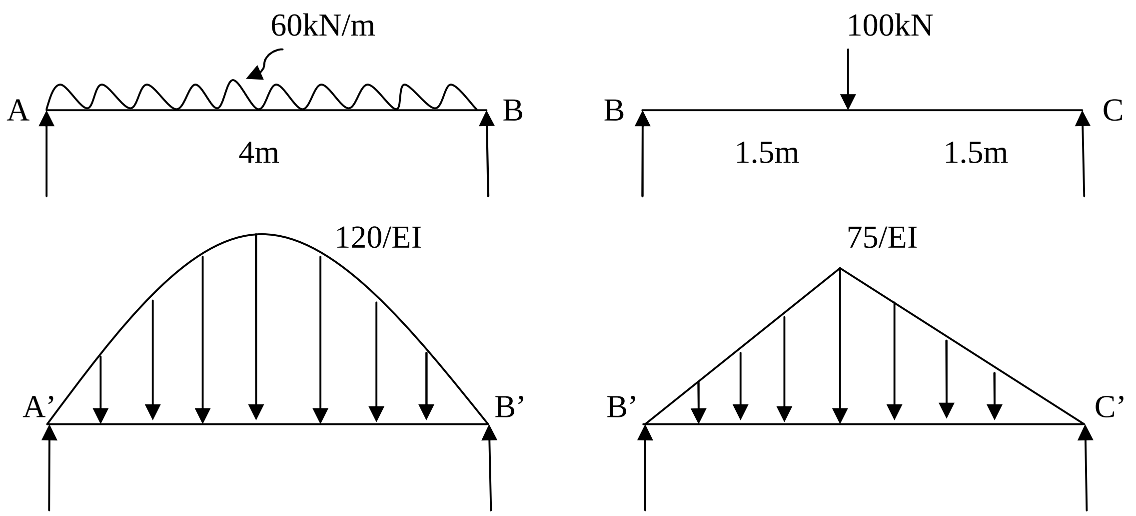
1. Analyse the continuous beam shown in the figure by flexibility matrix method, draw BMD



Static Indeterminacy SI = 2 (M_A and M_B)

M_A and M_B are the redundant

Let us remove the redundant to get primary determinate structure



$$[\Delta_L] = \begin{pmatrix} \Delta_{1L} \\ \\ \Delta_{2L} \end{pmatrix}$$

Δ_{1L} = Rotation at A = SF at A'

$$\Delta_{1L} = ½ \,[2/3 \text{ X } 4 \text{ X } \frac{120}{EI}]$$

$$\Delta_{1L} = \frac{160}{EI}$$

Δ_{2L} = Rotation at A = SF at B'

$= V_{B1}' + V_{B2}'$

$$\Delta_{2L} = ½ \,[2/3 \text{ X } 4 \text{ X } \frac{120}{EI}] + ½ \,[1/2 \text{ X } 3 \text{ X } \frac{75}{EI}]$$

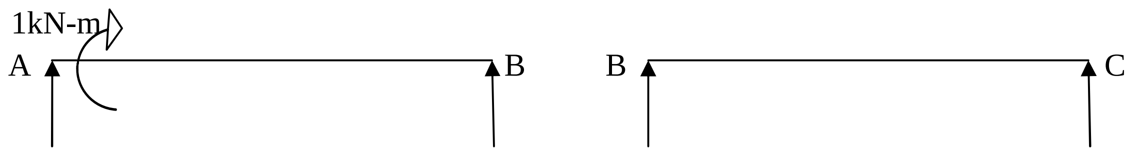
$$\Delta_{2L} = \frac{216.25}{EI}$$

$$[\Delta_L] = \frac{1}{EI}\begin{pmatrix} 160 \\ \\ 216.25 \end{pmatrix}$$

Note: The rotation due to sagging is taken as positive. The moments producing due to sagging are also taken as positive.

<u>To get Flexibility Matrix</u>

Apply unit moment to joint A

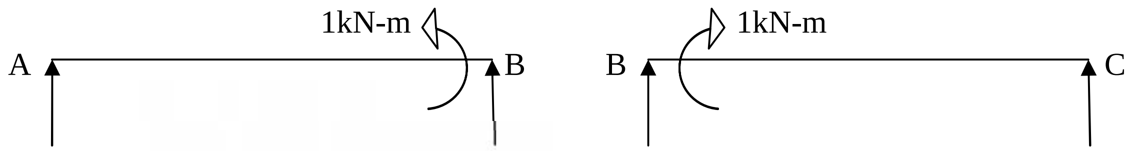


$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \\ \delta_{21} & \delta_{22} \end{pmatrix}$$

$$\delta_{11} = \frac{ml}{3EI} = \frac{1\ X\ 4}{3EI} = \frac{1.33}{EI}$$

$$\delta_{21} = \frac{ml}{6EI} = \frac{1\ X\ 4}{6EI} = \frac{0.67}{EI}$$

Apply unit moment to joint A



$$\delta_{12} = \frac{ml}{6EI} = \frac{1\ X\ 4}{6EI} = \frac{0.67}{EI}$$

$$\delta_{22} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1\ X\ 4}{3EI} + \frac{1\ X\ 3}{EI} = \frac{2.33}{EI}$$

$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} = \frac{1}{EI}\begin{pmatrix} 1.33 & 0.67 \\ 0.67 & 1.33 \end{pmatrix}$$

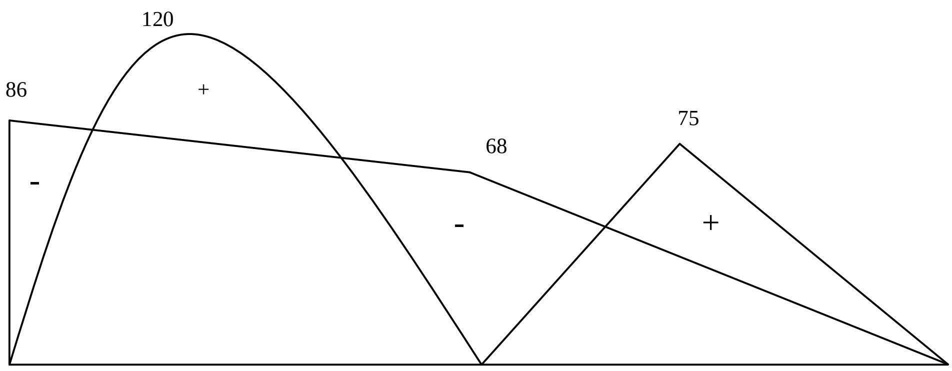
Apply the flexibility equation

$$[P] = [F]^{-1}\{[\Delta] - [\Delta_L]\}$$

$$[\Delta] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

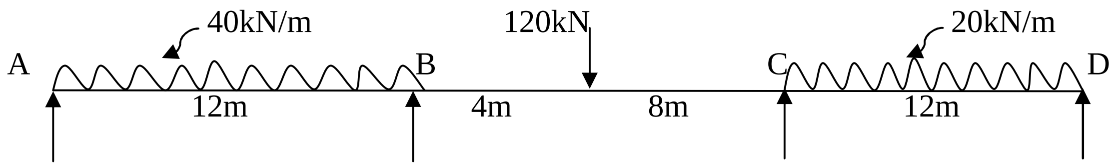
$$[P] = EI \begin{pmatrix} 1.33 & 0.67 \\ 0.67 & 1.33 \end{pmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \frac{1}{EI} \begin{bmatrix} 160 \\ 216.25 \end{bmatrix} \right\}$$

$$[P] = \begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} -86.00 \\ -68.08 \end{bmatrix} \text{kN-m}$$

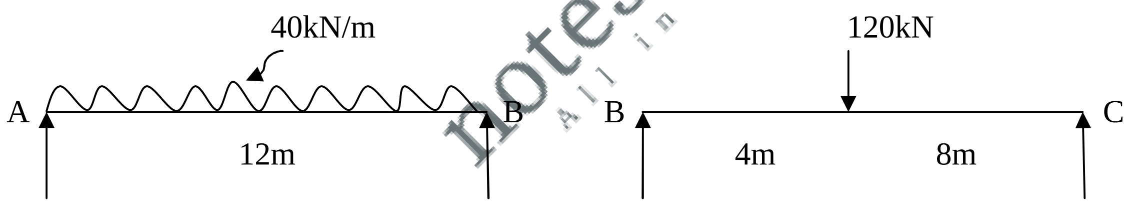


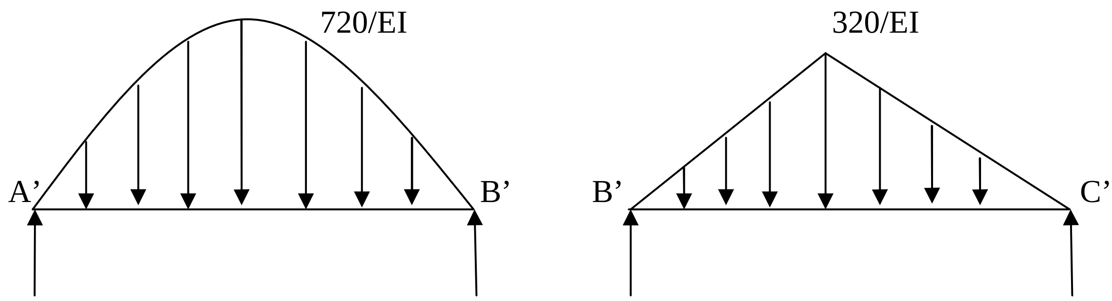
BMD

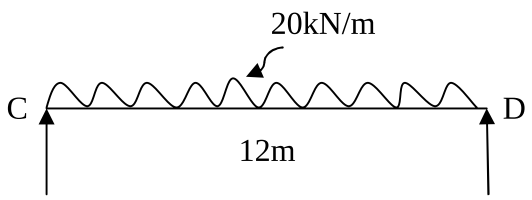
2. Analyse the continuous beam shown in the figure by flexibility matrix method, draw BMD

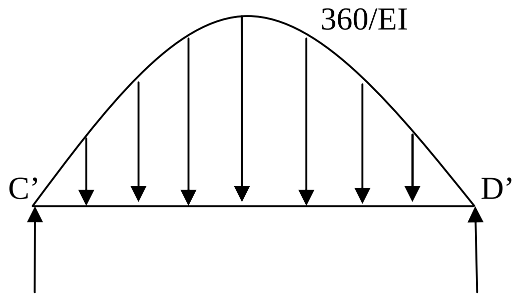


Static Indeterminacy SI = 2 (M_B and M_C)
M_B and M_C are the redundant
Let us remove the redundant to get primary determinate structure









$$[\Delta_L] = \begin{pmatrix} \Delta_{1L} \\ \Delta_{2L} \end{pmatrix}$$

Δ_{1L} = Rotation at B = SF at B’

$= V_{B1}' + V_{B2}'$

$$\Delta_{1L} = \frac{3946.67}{EI}$$

Δ_{2L} = Rotation at C = SF at C’

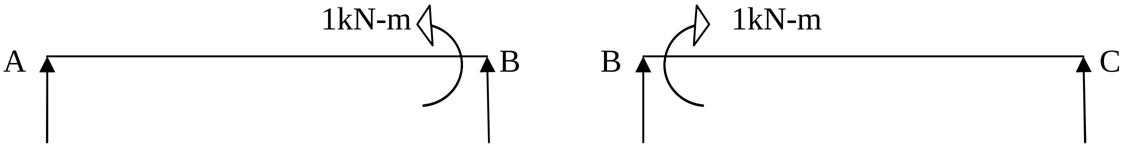
$= V_{C1}' + V_{C2}'$

$$\Delta_{2L} = \frac{2293.33}{EI}$$

$$[\Delta_L] = \frac{1}{EI}\begin{pmatrix} 3946.67 \\ 2293.33 \end{pmatrix}$$

<u>To get Flexibility Matrix</u>

Apply unit moment to joint A



$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}$$

$$\delta_{11} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1\,X\,12}{3EI} + \frac{1\,X\,12}{3EI} = \frac{8}{EI}$$

$$\delta_{21} = \frac{ml}{6EI} = \frac{1\,X\,12}{6EI} = \frac{2}{EI}$$

Apply unit moment to joint A

$$\delta_{12} = \frac{ml}{6EI} = \frac{1\,X\,12}{6EI} = \frac{2}{EI}$$

$$\delta_{22} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1\,X\,12}{3EI} + \frac{1\,X\,12}{EI} = \frac{8}{EI}$$

$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} = \frac{1}{EI} \begin{pmatrix} 8 & 2 \\ 2 & 8 \end{pmatrix}$$

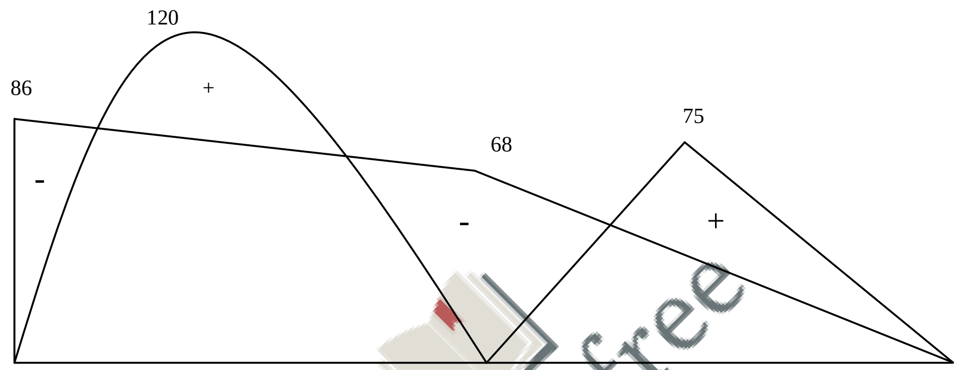
Apply the flexibility equation

$$[P] = [F]^{-1}\{[\Delta] - [\Delta_L]\}$$

$$[\Delta] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$[P] = EI \begin{pmatrix} 8 & 2 \\ 2 & 8 \end{pmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \frac{1}{EI} \begin{pmatrix} 3946 \\ 2293 \end{pmatrix} \right\}$$

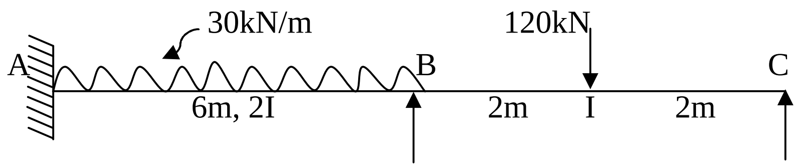
$$[P] = \begin{pmatrix} M_{AB} \\ M_{BA} \end{pmatrix} = \begin{pmatrix} -449.97 \\ -174.22 \end{pmatrix} \text{kN-m}$$



BMD

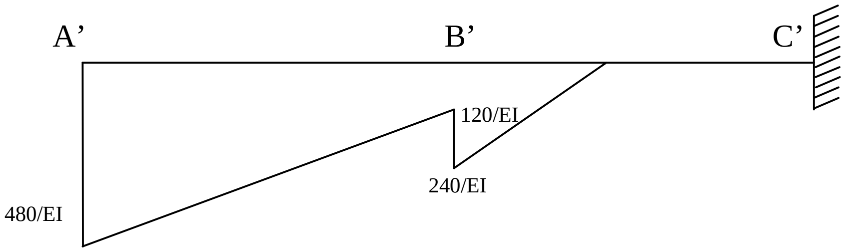
<u>**SINKING OF SUPPORT**</u>

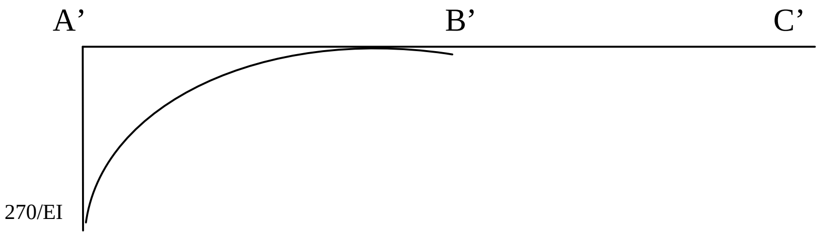
1. Analyse the continuous beam by flexibility method, support B sinks by 5mm. Sketch the BMD and EC given EI = 15 X 10^3 kN-m^2



NOTE: In this case of example with sinking of supports, the redundant should be selected as the vertical reaction.

Static indeterminacy is equal to 2. Let V_B and V_C be the redundant, remove the redundant to get the primary structure.





$$[\Delta_L] = \begin{pmatrix} \Delta_{1L} \\ \Delta_{2L} \end{pmatrix}$$

Δ_{1L} = Displacement at B in primary determinate structure = BM at B' in conjugate beam

$$\Delta_{1L} = [\tfrac{1}{2} X\ 6\ X\ \tfrac{360}{EI}\ X\ (2/3\ X\ 6)\ ] + (6\ X\ \tfrac{120}{EI}\ X\ 6/2) + [\tfrac{1}{3}\ X\ 6\ X\ \tfrac{270}{EI}\ X\ (3/4\ X\ 6)\ ]$$

$$\Delta_{1L} = \frac{8910}{EI}$$

Δ_{2L} = Displacement at C in primary determinate structure = BM at C' in conjugate beam

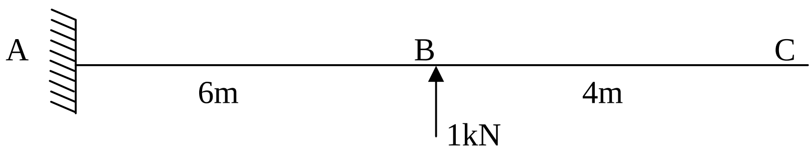
$$\Delta_{2L} = [\tfrac{1}{2}\ X\ 6\ X\ \tfrac{360}{EI}\ X\ (2/3\ X\ 6 + 4)\ ] + (6\ X\ \tfrac{120}{EI}\ X\ 6/2 + 4) + [\tfrac{1}{3}\ X\ 6\ X\ \tfrac{270}{EI}\ X\ (3/4\ X\ 6 + 4)\ ]$$

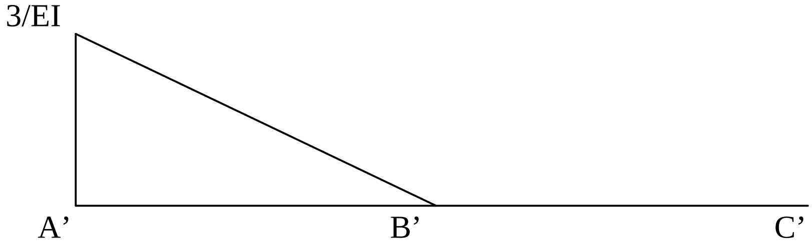
$$\Delta_{2L} = \frac{19070}{EI}$$

$$[\Delta_L] = \frac{1}{EI}\begin{pmatrix} 8910 \\ 19070 \end{pmatrix}$$

To get Flexibility Matrix

Apply unit Load at B



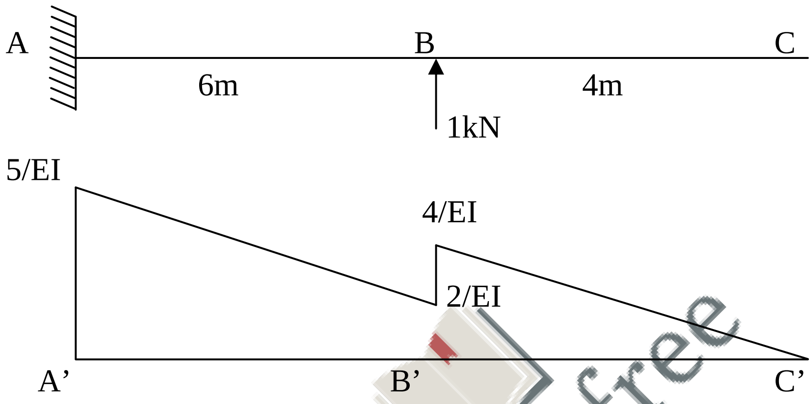


$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}$$

$$\delta_{11} = -\frac{1}{2} \text{ X } 6 \text{ X } \frac{3}{EI} \text{ X } (2/3 \text{ X } 6) = \frac{-36}{EI}$$

$$\delta_{21} = -\frac{1}{2} \text{ X } 6 \text{ X } \frac{3}{EI} \text{ X } (2/3 \text{ X } 6 + 4) = \frac{-72}{EI}$$

Apply unit load at C



$$\delta_{12} = -\frac{1}{2} \text{ X } 6 \text{ X } \frac{3}{EI} \text{ X } (2/3 \text{ X } 6) - [6 \text{ X } \frac{2}{EI} \text{ X } (6/2)] = \frac{-72}{EI}$$

$$\delta_{22} = -\frac{1}{2} \text{ X } 6 \text{ X } \frac{3}{EI} \text{ X } (2/3 \text{ X } 6 + 4) - [6 \text{ X } \frac{2}{EI} \text{ X } (6/2 + 4)] - \frac{1}{2} \text{ X } 4 \text{ X } \frac{4}{EI} \text{ X } (2/3 \text{ X } 4) = \frac{-177.33}{EI}$$

$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} = \frac{1}{EI} \begin{pmatrix} -36 & -72 \\ -72 & -177.33 \end{pmatrix}$$

Apply the flexibility equation

$$[P] = [F]^{-1}\{[\Delta] - [\Delta_L]\}$$

$$[\Delta] = \begin{pmatrix} 0.005 \\ 0 \end{pmatrix}$$

$$[P] = EI\begin{pmatrix} -36 & -72 \\ -72 & -177.33 \end{pmatrix}^{-1} \left\{ \begin{pmatrix} 0.005 \\ 0 \end{pmatrix} - \frac{1}{EI}\begin{pmatrix} 8910 \\ 19070 \end{pmatrix} \right\}$$

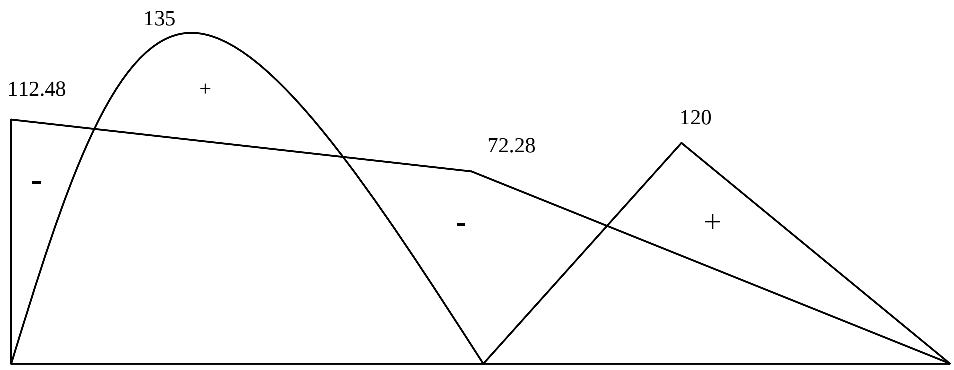
$$[P] = \begin{pmatrix} V_B \\ V_C \end{pmatrix} = \begin{pmatrix} 161.43 \\ 41.98 \end{pmatrix} \text{ kN-m}$$

<u>Support Reaction</u>

V_A = 96.64kN, V_{B1} = 83.36kN, V_{B2} = 78.07kN, V_C = 41.98kN

$V_B = V_{B1} + V_{B2}$ = 161.43kN

$$\begin{pmatrix} M_A \\ M_B \end{pmatrix} = \begin{pmatrix} 112.48 \\ 72.28 \end{pmatrix} \text{ kN-m}$$



BMD

MODULE-5

STIFFNESS MATRIX METHOD

The systematic development of slope deflection method in the matrix form has lead to Stiffness matrix method. The method is also called Displacement method. Since the basic unknowns are the displacement at the joint.

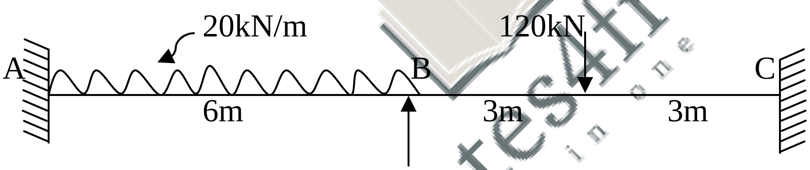
The stiffness matrix equation is given by

$[\Delta]\ [K] = \{[P] - [P_L]\}$

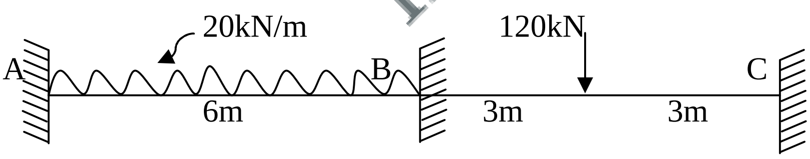
$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$

Where,
[P] = Redundant in matrix form
[F] = Stiffness matrix
[P] = Final force at the joints in matrix form
$[P_L]$= force at the joints due to applied load in matrix form

1. Analyse the continuous beam by Stiffness method Sketch the BMD



Kinematic Indeterminacy KI = 1 (θ_B)

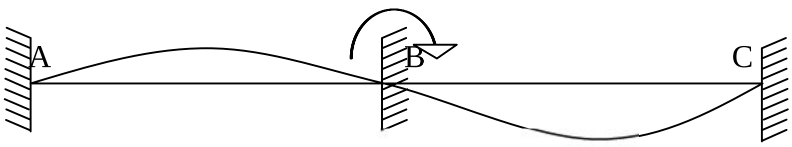


$[P_L] = M_{FBA} + M_{FBC}$

$$= \frac{wl^2}{12} + \left(-\frac{wl}{8}\right) = \frac{20\,X6^2}{12} - \frac{120\,X6}{8}$$

$[P_L]$ = -30kN-m

Apply unit displacement at joint B.



$$[K] = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} = \frac{4EI}{6} + \frac{4EI}{6} = 1.33EI \qquad (\theta=1)$$

By condition of equilibrium at joint B

$$[P] = 0$$

$$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$$

$$= \frac{1}{K}\{[P] - [P_L]\}$$

$$\theta_B = \frac{1}{1.33EI}\{[0] - [-30]\} = \frac{22.56}{EI}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= -60 + \frac{2\,EI}{6}\left(2\cancelto{0}{\theta_A} + \frac{22.5}{EI}\right)$$ ($\theta_A = 0$ due to fixity at support A)

$$M_{AB} = -52.5\text{kN-m}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A)$$

$$= 60 + \frac{2\,EI}{6}\left(2 \text{ X } \frac{22.5}{EI} + \cancelto{0}{\theta_A}\right)$$

$$M_{BA} = 75.04\text{kN-m}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

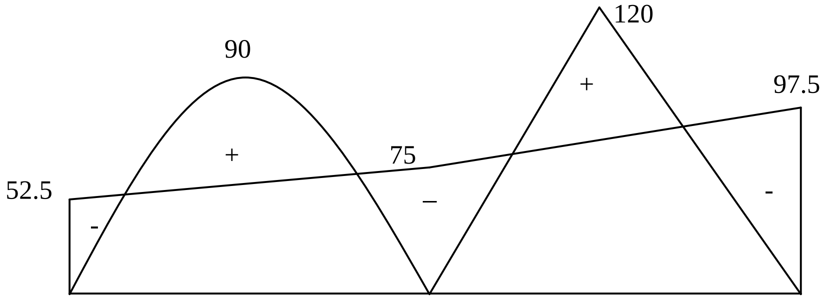
$$= -90 + \frac{2EI}{6}\left(2 \text{ X } \frac{22.5}{EI} + 0\right)$$

$$M_{BC} = -75\text{kN-m}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}(2\theta_C + \theta_B)$$

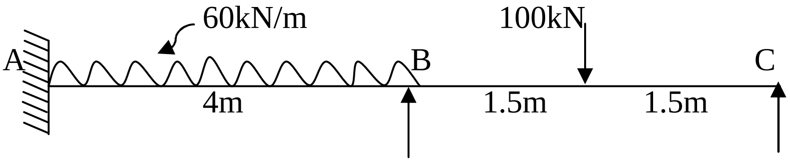
$$= 90 + \frac{2EI}{6}\left(0 + \frac{22.5}{EI}\right)$$

$$M_{CB} = 97.52\text{kN-m}$$

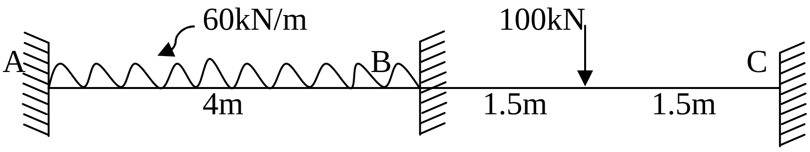


BMD

2. Analyse the continuous beam by Stiffness method Sketch the BMD



Kinematic Indeterminacy KI = 2 (θ_B & θ_C)



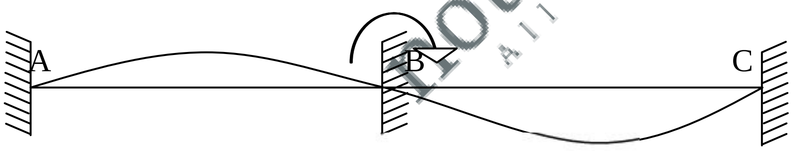
$[P_{1L}] = M_{FBA} + M_{FBC}$

$$= \frac{wl^2}{12} + \left(-\frac{wl}{8}\right) = \frac{60\,X\,4^2}{12} - \frac{100\,X3}{8} = 42.5\text{kN-m}$$

$[P_{2L}] = M_{FCB} = \frac{wl}{8} = \frac{100\,X\,3}{8} = 37.5\text{kN-m}$

$$[P_L] = \begin{pmatrix} P_{1L} \\ P_{2L} \end{pmatrix} = \begin{pmatrix} 42.5 \\ 37.5 \end{pmatrix} \text{kN-m}$$

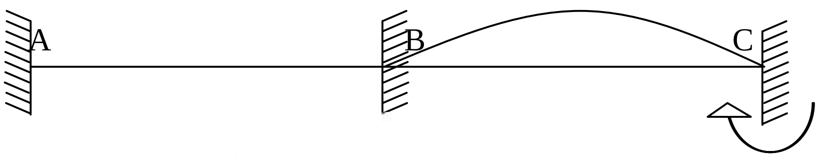
Apply unit displacement at joint B.



$K_{11} = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} = \frac{4EI}{4} + \frac{4EI}{3} = 2.33\text{EI}$ $\quad(\theta=1)$

$K_{21} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67\text{EI}$

Apply unit displacement at joint B.



$K_{12} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67\text{EI}$

$K_{22} = \frac{4EI\theta}{l} = \frac{4EI}{3} = 1.33\text{EI}$

By condition of equilibrium at joint B

$$[P] = 0$$

$$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$$

$$[\Delta] = \frac{1}{EI}\begin{pmatrix} 2.33 & 0.67 \\ 0.67 & 1.33 \end{pmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 42.5 \\ 37.5 \end{bmatrix} \right\}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -11.88 \\ -22.19 \end{bmatrix}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= -80 + \frac{2\,EI}{4}\left(2\cancelto{0}{\theta_A} - \frac{11.88}{EI}\right)$$ ($\theta_A = 0$ due to fixity at support A)

M_{AB} = -85.94kN-m

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A)$$

$$= 80 + \frac{2\,EI}{4}\left(2 \text{ X } \frac{-11.88}{EI} + \cancelto{0}{\theta_A}\right)$$

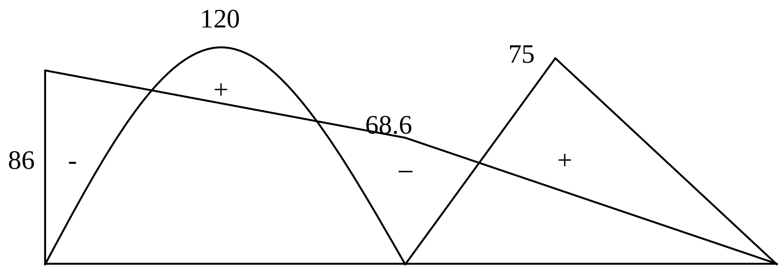
M_{BA} = 68.12kN-m

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

$$= -37.5 + \frac{2EI}{6}\left(2 \text{ X } \frac{-11.88}{EI} + \frac{-22.9}{EI}\right)$$

M_{BC} = -68.6kN-m

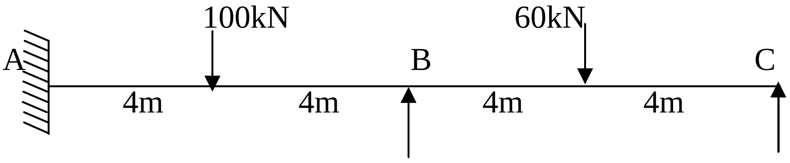
M_{CB} = 0



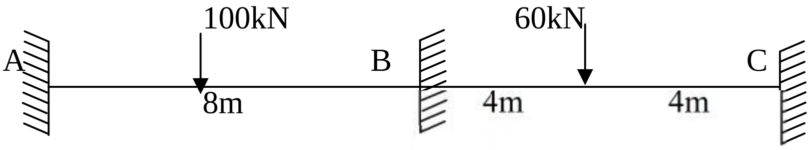
BMD

Sinking of support

1. Analyse the continuous beam shown in figure by stiffness method. Support B sinks by 300/EI units and support C sinks by 200/EI units



Kinematic Indeterminacy KI = 2 (θ_B & θ_C)



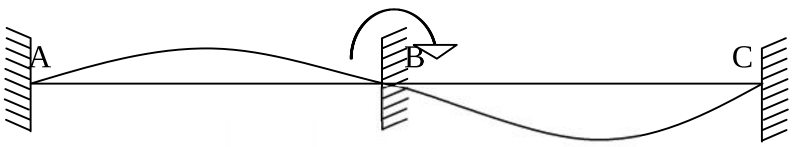
$$[P_{1L}] = M_{FBA} + M_{FBC} - \frac{6EI\delta}{l^2} - \frac{6EI\delta}{l^2}$$

$$= \frac{100 \ X\ 8}{8} - \frac{60\ X 8}{8} - \frac{6\ X\ 300}{8^2} + \frac{6\ X\ 100}{8^2} = 21.25\text{kN-m}$$

$$[P_{2L}] = M_{FCB} - \frac{6EI\delta}{l^2} = \frac{60\ X 8}{8} + \frac{6\ X\ 100}{8^2} = 69.38\text{kN-m}$$

$$[P_L] = \begin{pmatrix} P_{1L} \\ P_{2L} \end{pmatrix} = \begin{pmatrix} 21.25 \\ 69.38 \end{pmatrix} \text{kN-m}$$

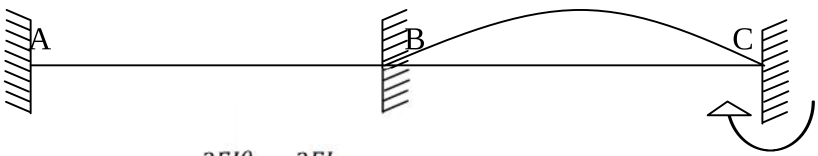
Apply unit displacement at joint B.



$$K_{11} = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} = \frac{4EI}{8} + \frac{4EI}{8} = EI \qquad (\theta=1)$$

$$K_{21} = \frac{2EI\theta}{8} = \frac{2EI}{3} = 0.25EI$$

Apply unit displacement at joint B.



$$K_{12} = \frac{2EI\theta}{l} = \frac{2EI}{8} = 0.25EI$$

$K_{22} = \frac{4EI\theta}{l} = \frac{4EI}{8} = 0.50EI$

By condition of equilibrium at joint B

$[P] = 0$

$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$

$$[\Delta] = \frac{1}{EI}\begin{pmatrix} 1 & 0.25 \\ 0.25 & 0.50 \end{pmatrix}^{-1} \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 21.25 \\ 69.38 \end{pmatrix} \right\}$$

$$\begin{pmatrix} \theta_B \\ \theta_C \end{pmatrix} = \frac{1}{EI} \begin{pmatrix} 15.36 \\ -146.44 \end{pmatrix}$$

Slope deflection equation

$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B - \frac{3(+\delta)}{l})$

$= -100 + \frac{2\,EI}{8}(2\cancelto{0}{\theta_A} + \frac{15.36}{EI} - \frac{3EI(300/EI)}{8})$ ($\theta_A = 0$ due to fixity at support A)

$M_{AB} = -124.29$kN-m

$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_A + \theta_B - \frac{3\delta}{l})$

$= 100 + \frac{2\,EI}{8}(\cancelto{0}{\theta_A} + 2\,X\,\frac{15.36}{EI} - \frac{3EI(300/EI)}{8})$

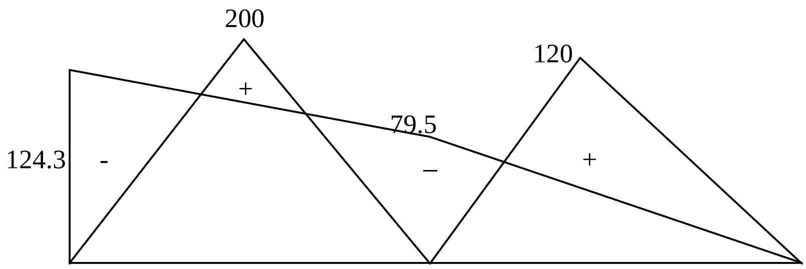
$M_{BA} = 79.55$kN-m

$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C - \frac{3\delta}{l})$

$= -60 + \frac{2\,EI}{8}(2\,X\,\frac{15.36}{EI} + \frac{-146.44}{EI} - \frac{3EI(-100/EI)}{5})$

$M_{BC} = -79.55$kN-m

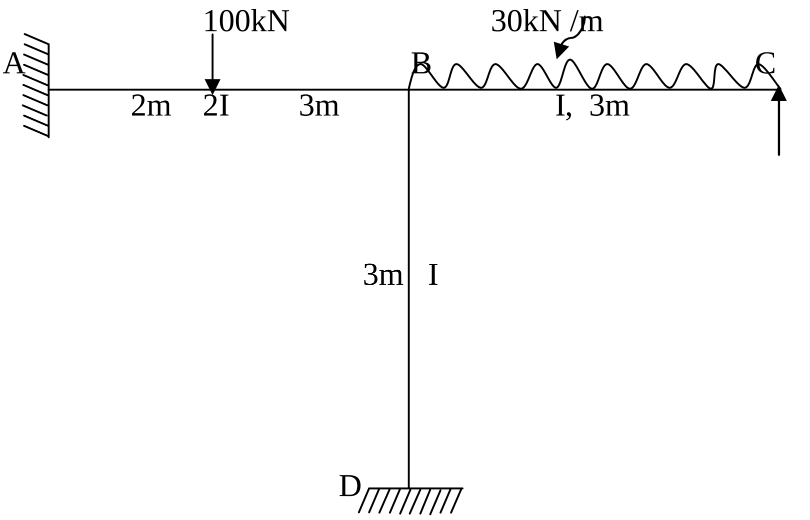
$M_{CB} = 0$



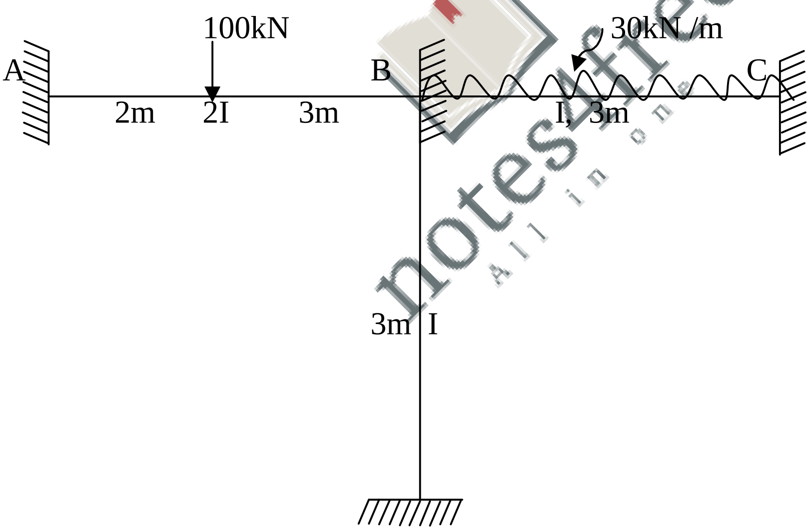
BMD

Analysis of frames

1. Analyse the frame by stiffness method



Kinematic Indeterminacy KI = 2 (θ_B & θ_C)



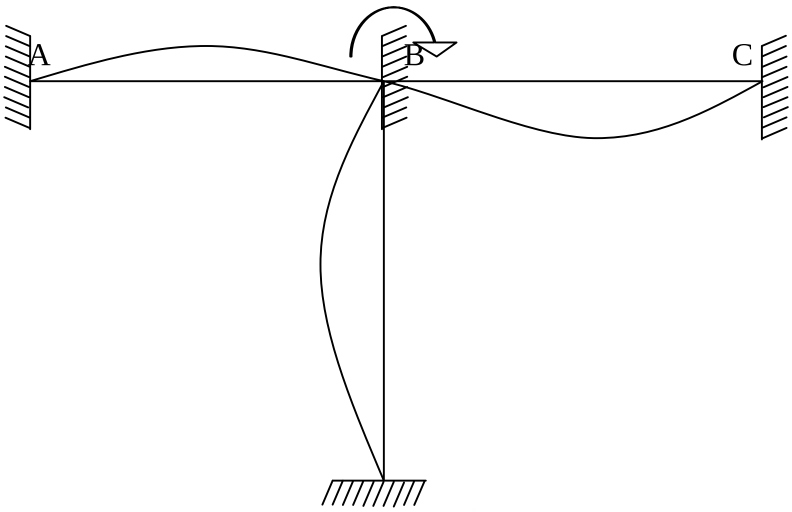
$[P_{1L}] = M_{FBA} - M_{FBC} + M_{FCD}$

$$= \frac{100 X 2 X 3^2}{5^2} - \frac{30 X 3^2}{12} = 25.5\text{kN-m}$$

$[P_{2L}] = M_{FCB} = \frac{30 X 3^2}{12} = 22.5\text{kN-m}$

$$[P_L] = \begin{pmatrix} P_{1L} \\ P_{2L} \end{pmatrix} = \begin{pmatrix} 25.5 \\ 22.5 \end{pmatrix} \text{kN-m}$$

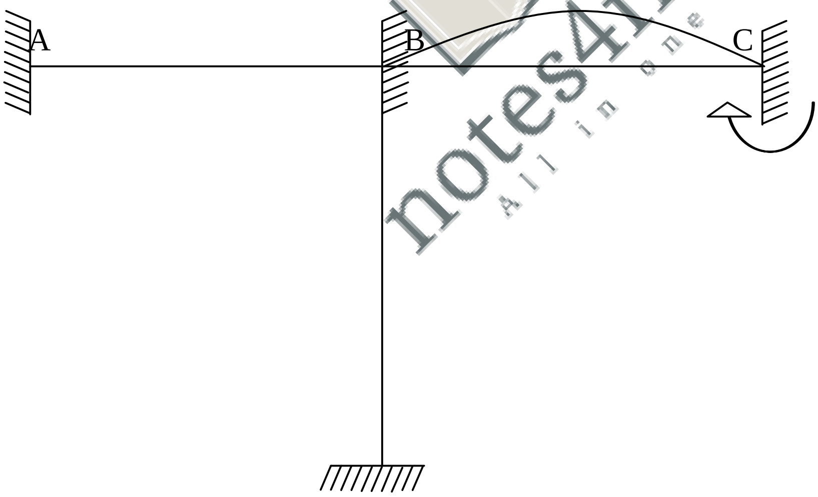
Apply unit displacement at joint B.



$$K_{11} = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} + \frac{4EI\theta}{l} = \frac{4\ X 2EI}{5} + \frac{4EI}{3} + \frac{4EI}{3} = 4.267EI \qquad (\theta=1)$$

$$K_{21} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI$$

Apply unit displacement at joint C.



$$K_{12} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI$$

$$K_{22} = \frac{4EI\theta}{l} = \frac{4EI}{3} = 1.33EI$$

By condition of equilibrium at joint B

$[P] = 0$

$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$

$$[\Delta] = \frac{1}{EI}\begin{pmatrix} 4.267 & 0.67 \\ 0.67 & 1.33 \end{pmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 25.5 \\ 22.5 \end{bmatrix} \right\}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -3.604 \\ -15.01 \end{bmatrix}$$

Slope deflection equation

$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$

$= -72 + \frac{2\,X\,2EI}{5}\left(2\cancelto{0}{\theta_A} + \frac{-3.604}{EI}\right)$ ($\theta_A = 0$ due to fixity at support A)

$M_{AB} = -74.88$kN-m

$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_A + \theta_B)$

$= 72 + \frac{2\,X\,2EI}{5}\left(\cancelto{0}{\theta_A} + 2\,X\,\frac{-3.604}{EI}\right)$

$M_{BA} = 42.23$kN-m

$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$

$= -22.5 + \frac{2\,EI}{3}\left(2\,X\,\frac{-3.604}{EI} + \frac{-15.1}{EI}\right)$

$M_{BC} = -37.37$kN-m

$M_{BD} = M_{FBD} + \frac{2EI}{l}(2\theta_B + \theta_D)$

$= 0 + \frac{2\,EI}{3}\left(2\,X\,\frac{-3.604}{EI} + 0\right)$

$M_{BD} = -4.81$kN-m

$M_{DB} = M_{FDB} + \frac{2EI}{l}(2\theta_D + \theta_B)$

$= 0 + \frac{2\,EI}{3}\left(2\,X 0 + \frac{-3.604}{EI}\right)$

$M_{DB} = -2.402$kN-m

$M_{CB} = 0$